

# QG Height Tendency

Arranged by Andrew Moore  
Material from Thomas Galarneau

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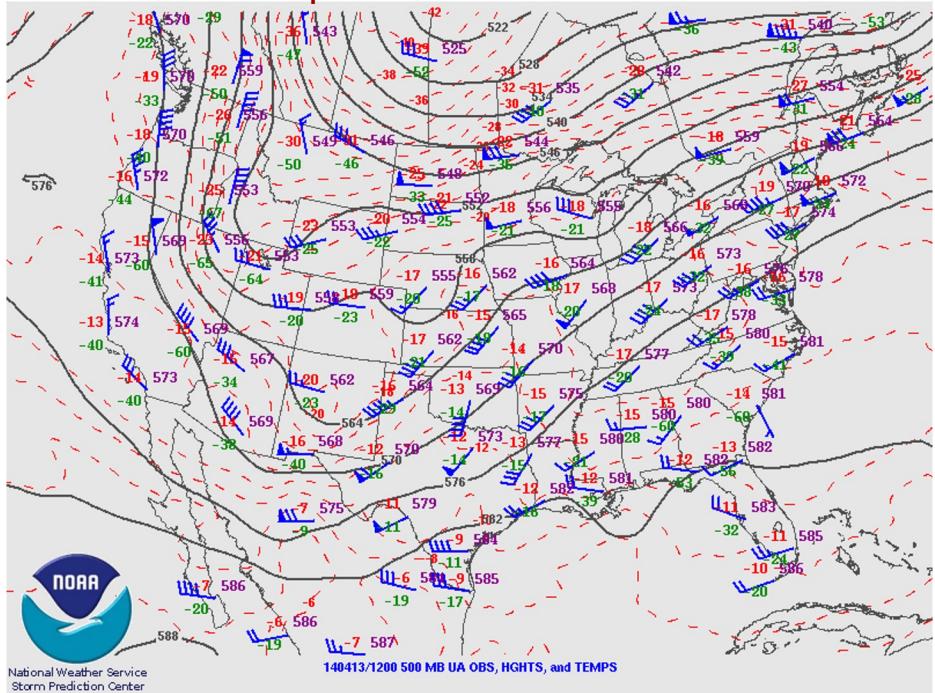
## Why do we care?

The QG height tendency equation allows us to anticipate:

- The evolution of upper air and surface patterns
- The evolution of certain severe weather parameters (e.g. shear, lift, etc...)

It is also relatively easy to use!

What will this pattern look like in 12-24 hours?



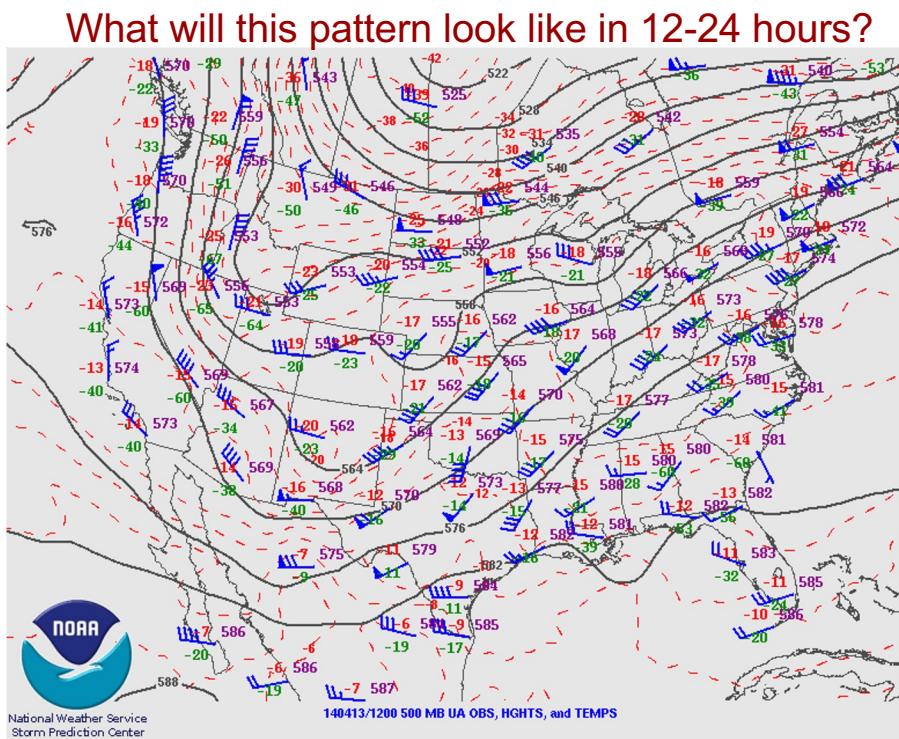
# QG Height Tendency

## A quick note:

There are alternatives to QG theory (for example, IPV theory), that will work just as well.

We will focus on QG theory in this class for two reasons:

- 1) It's easy to interpret from basic weather charts
- 2) Most U.S. weather entities use QG theory (not the case elsewhere...)



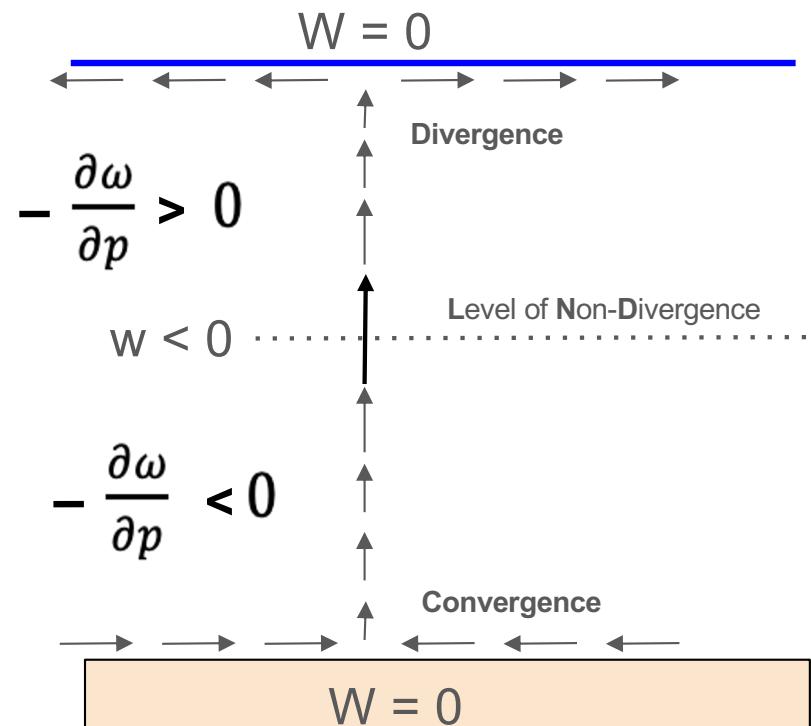
# Some Background Concepts

**Mass Continuity:**

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial \omega}{\partial p} = 0$$

We're going to assume:

- On synoptic scales, the troposphere is incompressible.
- Hydrostatic approximation applies
- Vertical velocity is zero at the surface and at the tropopause.
- **Because of mass continuity, any vertical motion is associated with horizontal convergence and divergence**



# Some Background Concepts

## 1.4 thermal wind balance

$$(1) u_g = -\frac{g}{f} \frac{\partial Z}{\partial y} \quad \text{geostrophic wind}$$

$$(2) \frac{\partial Z}{\partial p} = -\frac{RT}{gp} \quad \text{hypsometric eqn}$$

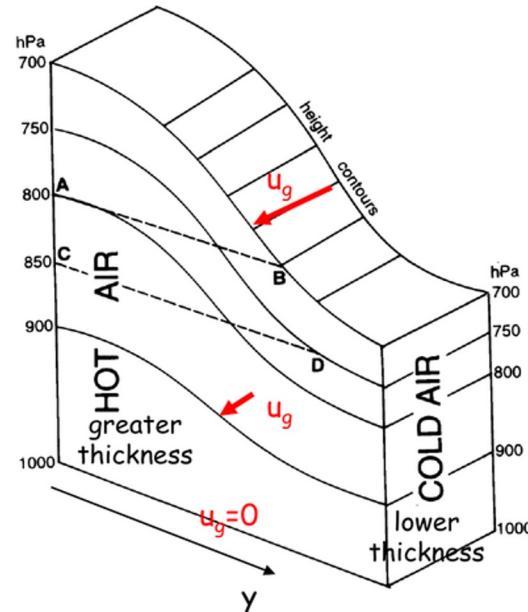
plug (2) into (1)

$$\begin{aligned} \frac{\partial u_g}{\partial p} &= \frac{g}{f} \frac{\partial (RT/gp)}{\partial y} \\ &= \frac{R}{fp} \frac{\partial T}{\partial y} \end{aligned}$$

finite difference expression:

$$\Delta u_g = \frac{R}{f} \frac{\Delta p}{p} \frac{\Delta \bar{T}}{\Delta y} \quad \text{this is the thermal wind: an increase in wind with height due to a temperature gradient}$$

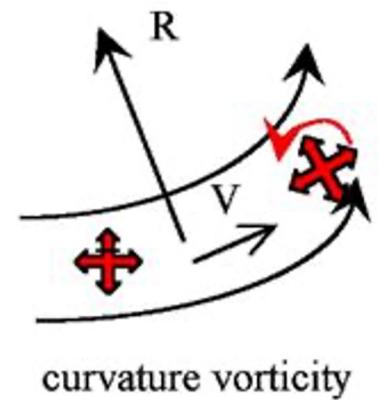
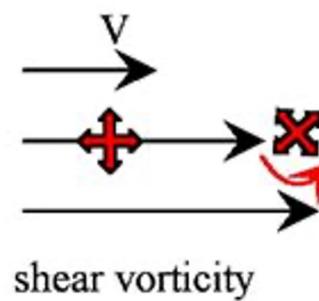
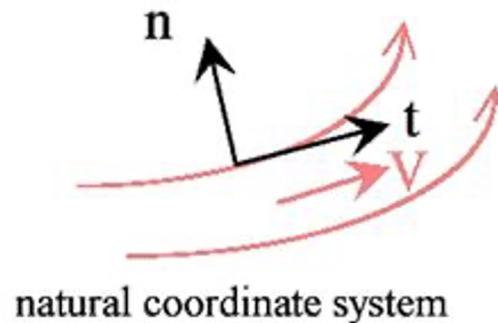
The thermal wind blows ccw around cold pools in the same way as the geostrophic wind blows ccw around lows. The thermal wind is proportional to the T gradient, while the geostrophic wind is proportional to the pressure (or height) gradient.



# Some Background Concepts

## Vorticity:

- Vorticity is the curl of the wind field
- Exists in all 3 dimensions, but for today we'll only consider the X/Y dimensions
- Vorticity can be generated by curvature in the flow and/or speed shear in the flow
- Vorticity is directly related to vertical motions and convergence/divergence due to the conservation of angular momentum and conservation of mass



# Physical Intuition

## Charles's Law

- The volume of a gas is directly proportional to the temperature of the gas at a constant pressure.
- If the gas heats up -> it expands!
- If the gas cools down -> it contracts!



Jacques Charles  
(1746-1823).



# Physical Intuition

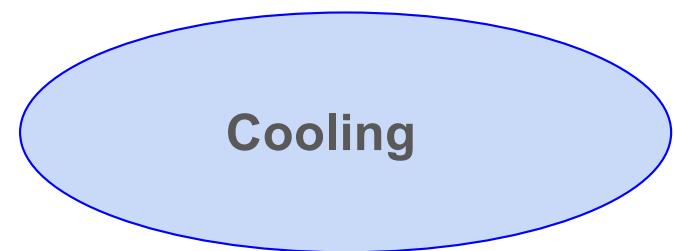


Consider a layer of the atmosphere in contact with the surface:

Warming the layer

Cooling the layer

Initial Height



The ground can't move - so the top of the layer must move!

# Physical Intuition



Consider a layer of the atmosphere in contact with the surface:

Warming the layer

Final Height

Initial Height

(Expands)

Cooling the layer

Final Height

(Contracts)

The ground can't move - so the top of the layer must move!

# Physical Intuition



Consider a layer of the atmosphere above the surface:

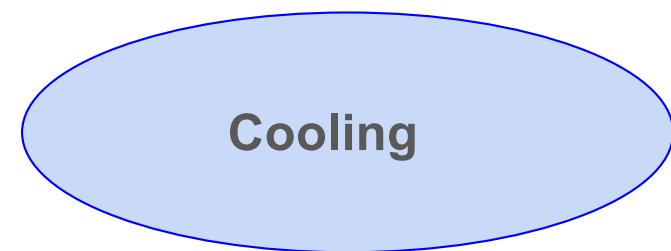
Warming the layer

Cooling the layer

Initial Height 1



Initial Height 2



# Physical Intuition

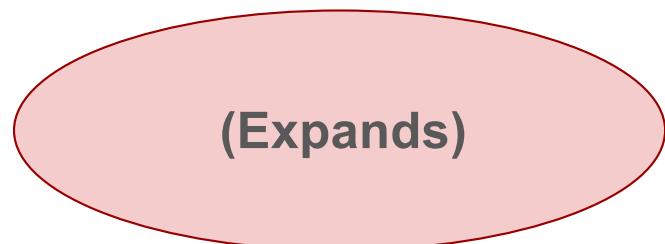


Consider a layer of the atmosphere above the surface:

Warming the layer

Final Height 1

Initial Height 1



Cooling the layer

Final Height 1

(Contracts)

Final Height 2

Initial Height 2

Final Height 2

# Physical Intuition

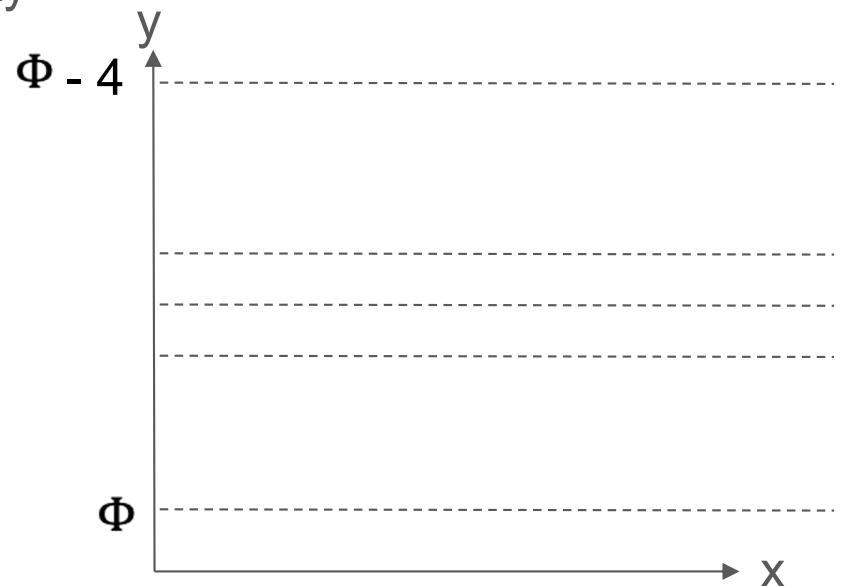
## Geostrophic Vorticity:

Plug in geostrophic wind balance into the vorticity equation.

You end up with a form of geostrophic vorticity that relates to the Laplacian of the geopotential height field.

Thus, if you locally change the vorticity at a location, you must also change the geopotential height (i.e. thickness) field!

Consider this geopotential height field. What would vorticity look like?



# Physical Intuition

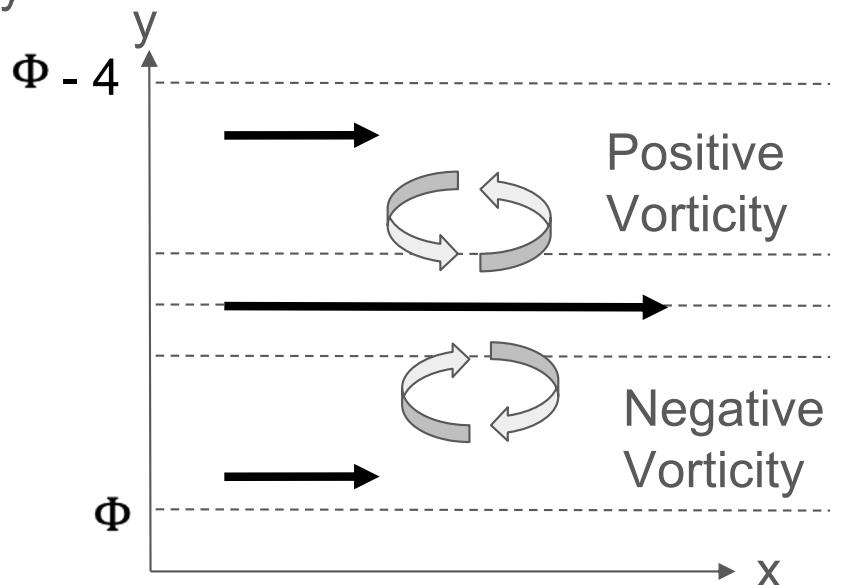
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Consider this geopotential height field. What would vorticity look like?



Use the thermal wind relation to confirm!

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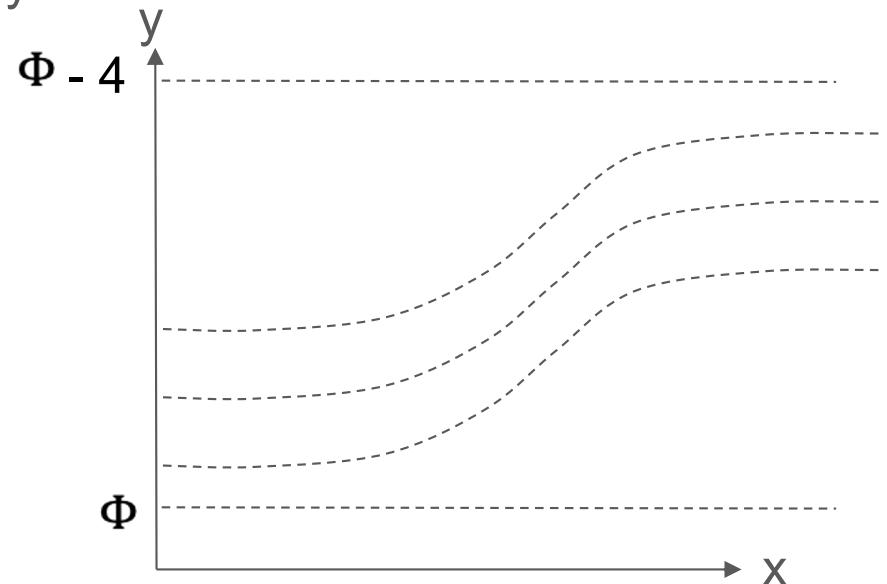
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Consider this geopotential height field. What would vorticity look like?



If we change the geopotential height field, how will the vorticity field change (and vise versa)?

# QG Vorticity and Thermo Equations

**QG vorticity equation**

$$\frac{d(\zeta_g + f)}{dt} = f_0 \frac{\partial \omega}{\partial p}$$

Rate of change of  
absolute vorticity

Vertical motion  
(or convergence/divergence)

See Bluestein Vol. 1  
page 329 for details

**QG thermodynamic equation**

$$\frac{dT}{dt} = \frac{p}{R_d} \sigma \omega$$

Rate of change of  
temperature

Vertical motion  
(or expansion/contraction  
from vertical motion)



# QG Height Tendency Equation

$$\left( \nabla_p^2 + \frac{f_0^2}{\sigma} \frac{\partial^2}{\partial p^2} \right) \chi = -f_0 \mathbf{V}_g \cdot \nabla_p \left( \frac{1}{f_0} \nabla_p^2 \Phi + f \right) - \frac{f_0^2}{\sigma} \frac{\partial}{\partial p} \left[ \mathbf{V}_g \cdot \nabla_p \left( -\frac{\partial \Phi}{\partial p} \right) \right] - \frac{\partial H}{\partial p}$$

2nd derivative operator

Absolute vorticity advection

Differential thermal advection

Diabatic heating

# QG Height Tendency Equation

$$\left(\nabla_p^2 + \frac{f_0^2}{\sigma} \frac{\partial^2}{\partial p^2}\right) \chi = -f_0 \mathbf{V}_g \cdot \nabla_p \left( \frac{1}{f_0} \nabla_p^2 \Phi + f \right) - \frac{f_0^2}{\sigma} \frac{\partial}{\partial p} \left[ \mathbf{V}_g \cdot \nabla_p \left( -\frac{\partial \Phi}{\partial p} \right) \right] - \frac{\partial H}{\partial p} \quad \chi \equiv \frac{\partial \Phi}{\partial t}$$

- Height change ( $A$ ) =  $B + C + D$
- Term B: advection of geostrophic absolute vorticity by the geostrophic wind
  - Cyclonic vorticity advection (CVA)  $\equiv$  height falls
  - Propagation mechanism for troughs and ridges
- Term C: differential advection of thickness by the geostrophic wind
  - Referred to as thermal advection or temperature advection
  - Heights rise above and fall below level of maximum warm advection
  - Heights fall above and rise below level of maximum cold advection
  - Amplification mechanism for troughs and ridges
- Term D: differential diabatic heating
  - Heights rise above and fall below level of maximum latent heating
  - Heights fall above and rise below level of maximum radiational cooling

# QG X Breakdown

$$\left( \nabla_p^2 + \frac{f_0^2}{\sigma} \frac{\partial^2}{\partial p^2} \right) \chi$$

2nd Derivative Operator:

On synoptic scales, we can roughly assume that the height field (and thus the height tendency field) is sinusoidal.

Take the second derivative of a sine function:

$$d(d(\sin(x)))$$

$$= d(\cos(x))$$

$$= -\sin(x)$$

**This assumption turns the 2nd derivative operation into a simple minus sign!**

# QG X Breakdown

$$-f_0 \mathbf{V}_g \cdot \nabla_p \left( \frac{1}{f_0} \nabla_p^2 \Phi + f \right)$$

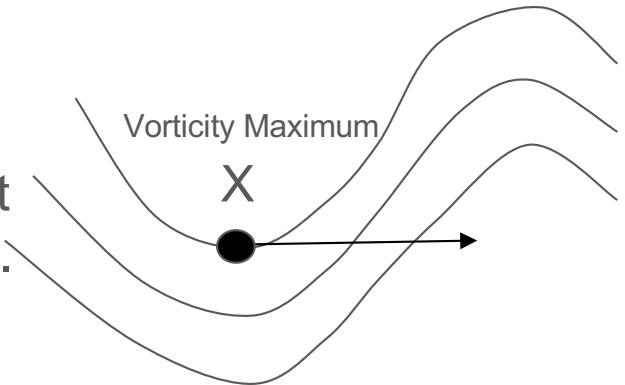
OR

$$-f_0 \mathbf{V}_g \cdot \nabla_p (\zeta_g + f)$$

## Advection of absolute vorticity:

- This considers both relative vorticity (related to the height field) and planetary vorticity (related to the Coriolis force).
- Here we see how moving the height field (more specifically, the Laplacian of the height field) can change the height field.

Consider this case:



$$\begin{aligned} Vg &= 0 \\ Ug &> 0 \end{aligned}$$

# QG X Breakdown

$$-f_0 \mathbf{V}_g \cdot \nabla_p \left( \frac{1}{f_0} \nabla_p^2 \Phi + f \right)$$

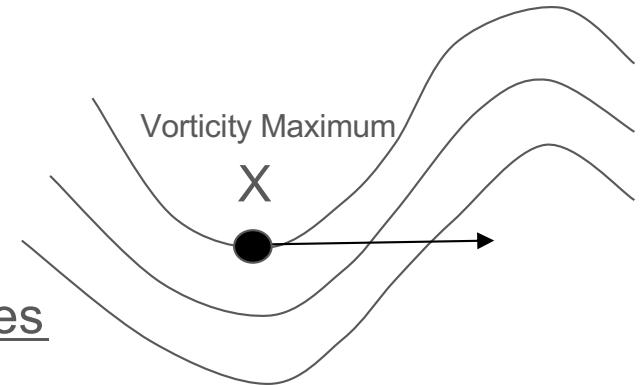
OR

$$-f_0 \mathbf{V}_g \cdot \nabla_p (\zeta_g + f)$$

Advection of absolute vorticity:

- Advecting cyclonic vorticity (CVA) leads to height falls
- Advecting anticyclonic vorticity (AVA) leads to height rises

Consider this case:



$$\begin{aligned} V_g &= 0 \\ U_g &> 0 \end{aligned}$$

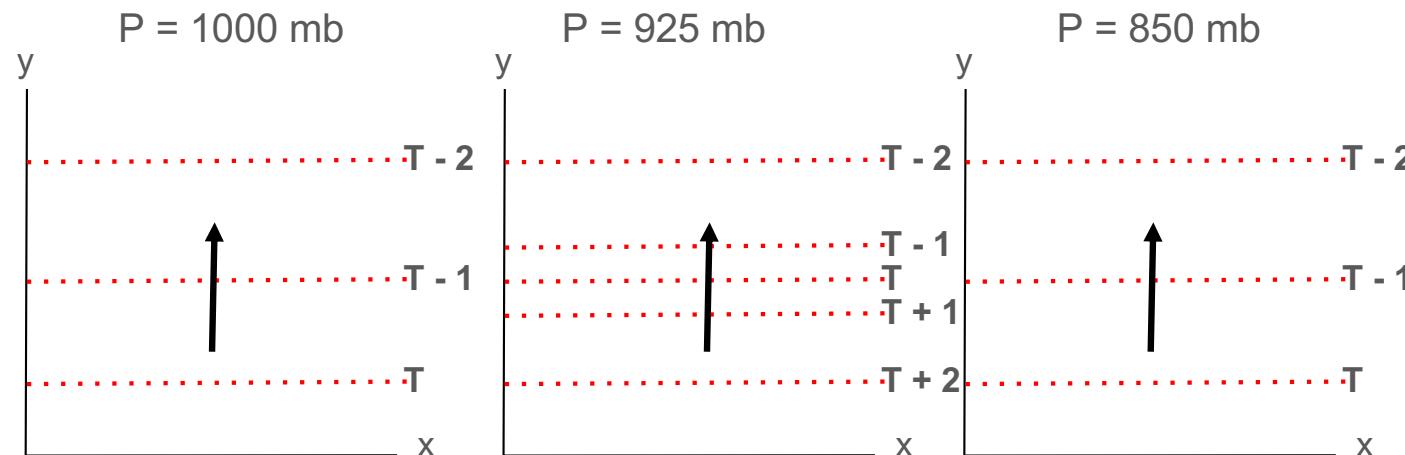
# QG X Breakdown

$$-\frac{f_0^2}{\sigma} \frac{\partial}{\partial p} \left[ \mathbf{v}_g \cdot \nabla_p \left( -\frac{\partial \Phi}{\partial p} \right) \right]$$

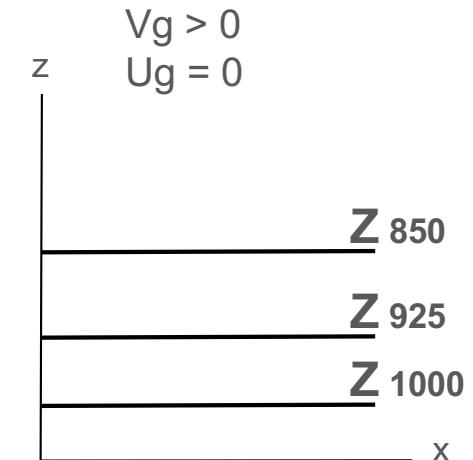
OR

$$-\frac{f_0^2}{\sigma} \frac{\partial}{\partial p} \left[ -\frac{R}{p} \mathbf{v}_g \cdot \nabla_p T \right]$$

## Differential Thermal Advection



Assume:



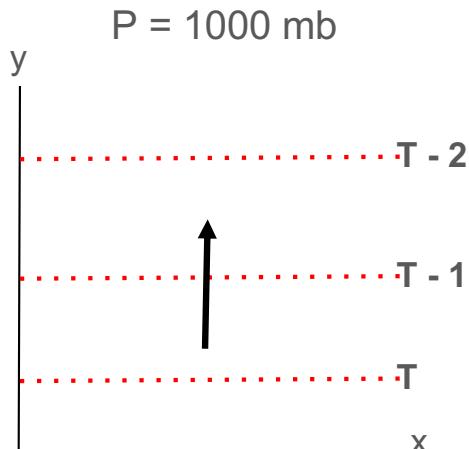
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OR

$$-\frac{f_0^2}{\sigma} \frac{\partial}{\partial p} \left[ -\frac{R}{p} \mathbf{v}_g \cdot \nabla_p T \right]$$

## Differential Thermal Advection



Consider only y component of thermal advection:

$$Vg > 0$$

$$dT/dy < 0$$

$$-Vg(dT/dy) > 0$$

Assume:

$$Vg > 0$$

$$Ug = 0$$

$x$

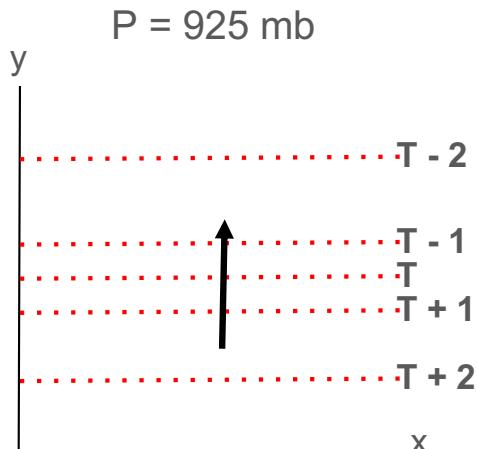
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## Differential Thermal Advection



Consider only y component of thermal advection:

$$Vg > 0$$

$$dT/dy \ll 0$$

$$-Vg(dT/dy) \gg 0$$

Assume:

$$Vg > 0$$

$$Ug = 0$$

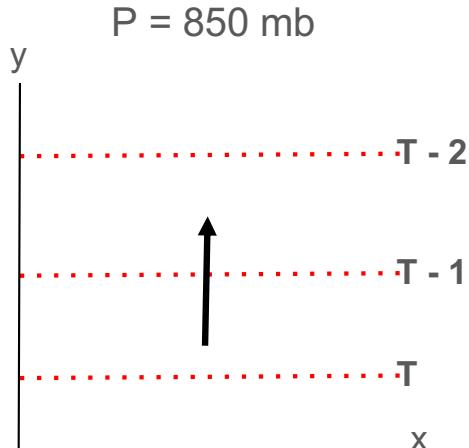
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$$\boxed{-\frac{f_0^2}{\sigma} \frac{\partial}{\partial p} \left[ \mathbf{v}_g \cdot \nabla_p \left( -\frac{\partial \Phi}{\partial p} \right) \right]}$$

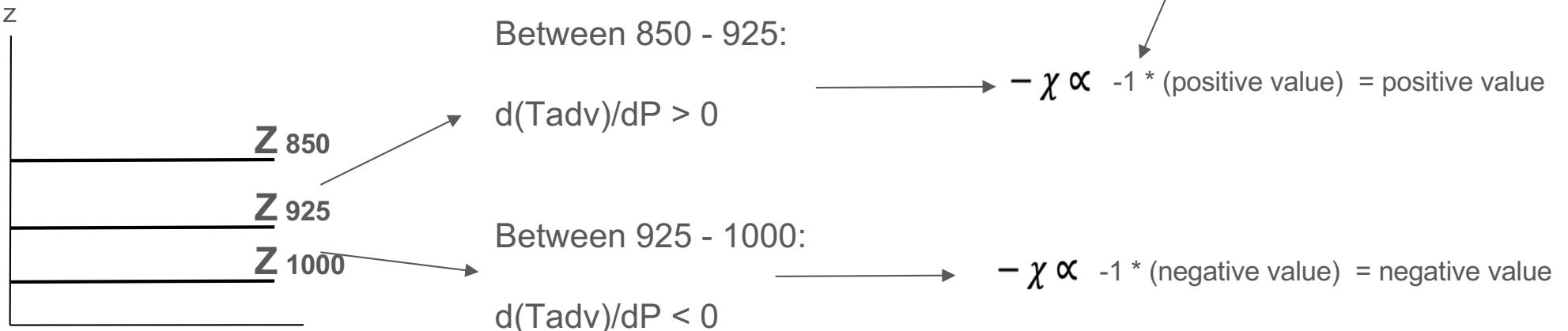
OR

$$-\frac{f_0^2}{\sigma} \frac{\partial}{\partial p} \left[ -\frac{R}{p} \mathbf{v}_g \cdot \nabla_p T \right]$$

Can't forget this -1!

## Differential Thermal Advection

Now consider differential portion between the pressure levels:



# QG X Breakdown

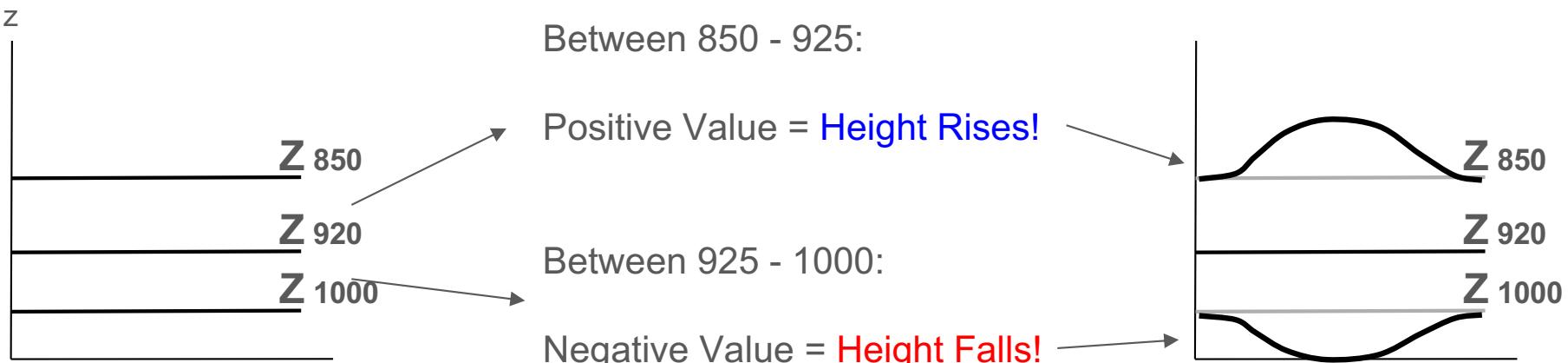
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## Differential Thermal Advection

Now consider differential portion between the pressure levels:



# QG X Breakdown

$$-\frac{f_0^2}{\sigma} \frac{\partial}{\partial p} \left[ \mathbf{v}_g \cdot \nabla_p \left( -\frac{\partial \Phi}{\partial p} \right) \right]$$

OR

$$-\frac{f_0^2}{\sigma} \frac{\partial}{\partial p} \left[ -\frac{R}{p} \mathbf{v}_g \cdot \nabla_p T \right]$$

## Differential Thermal Advection:

- Warm air advection at a given pressure level induces height rises above that level, and height falls below that level
- Cold air advection does the opposite: it cases height falls above the given pressure level, and height rises below.
- This ties back directly to Charles's Law!



# QG X Breakdown

$$-\frac{\partial H}{\partial p}$$

## Diabatic Heating:

- Similar in concept to differential thermal advection:  
Diabatic heating in a layer causes the layer to expand;  
Diabatic cooling in layer causes the layer to contract.
- Causes height rises (falls) above (below) the source of heating.
- Causes height rises (falls) below (above) the source of cooling
- Examples:
  - Latent heat release from large systems (hurricanes, large cyclones, etc...)
  - Mesohighs in the wake of strong MCSs

# QG X Breakdown



$$-\frac{\partial H}{\partial p}$$

Warming the layer

Final Height 1

Initial Height 1

Heating source

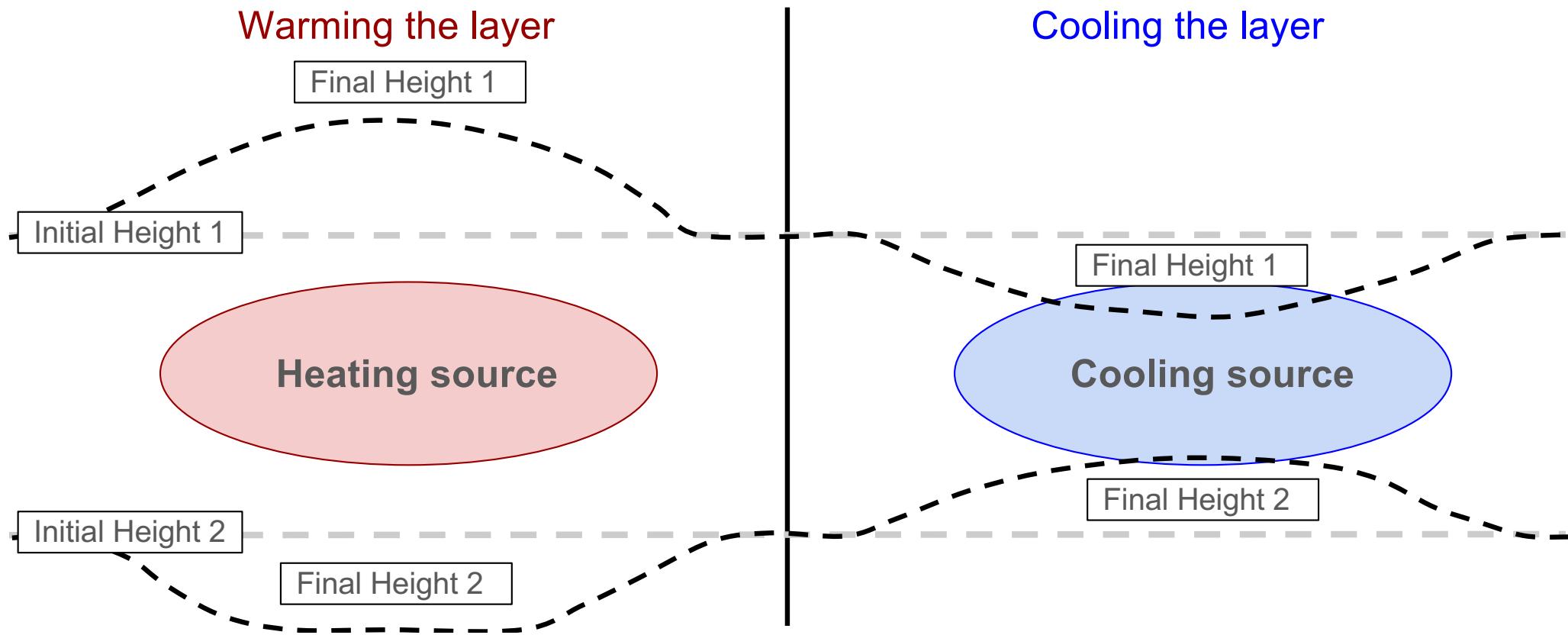
Cooling the layer

Final Height 1

Cooling source

Initial Height 2

Final Height 2



# QG X Application

$$\left(\nabla_p^2 + \frac{f_0^2}{\sigma} \frac{\partial^2}{\partial p^2}\right) \chi = -f_0 \mathbf{V}_g \cdot \nabla_p \left( \frac{1}{f_0} \nabla_p^2 \Phi + f \right) - \frac{f_0^2}{\sigma} \frac{\partial}{\partial p} \left[ \mathbf{V}_g \cdot \nabla_p \left( -\frac{\partial \Phi}{\partial p} \right) \right] - \frac{\partial H}{\partial p}$$

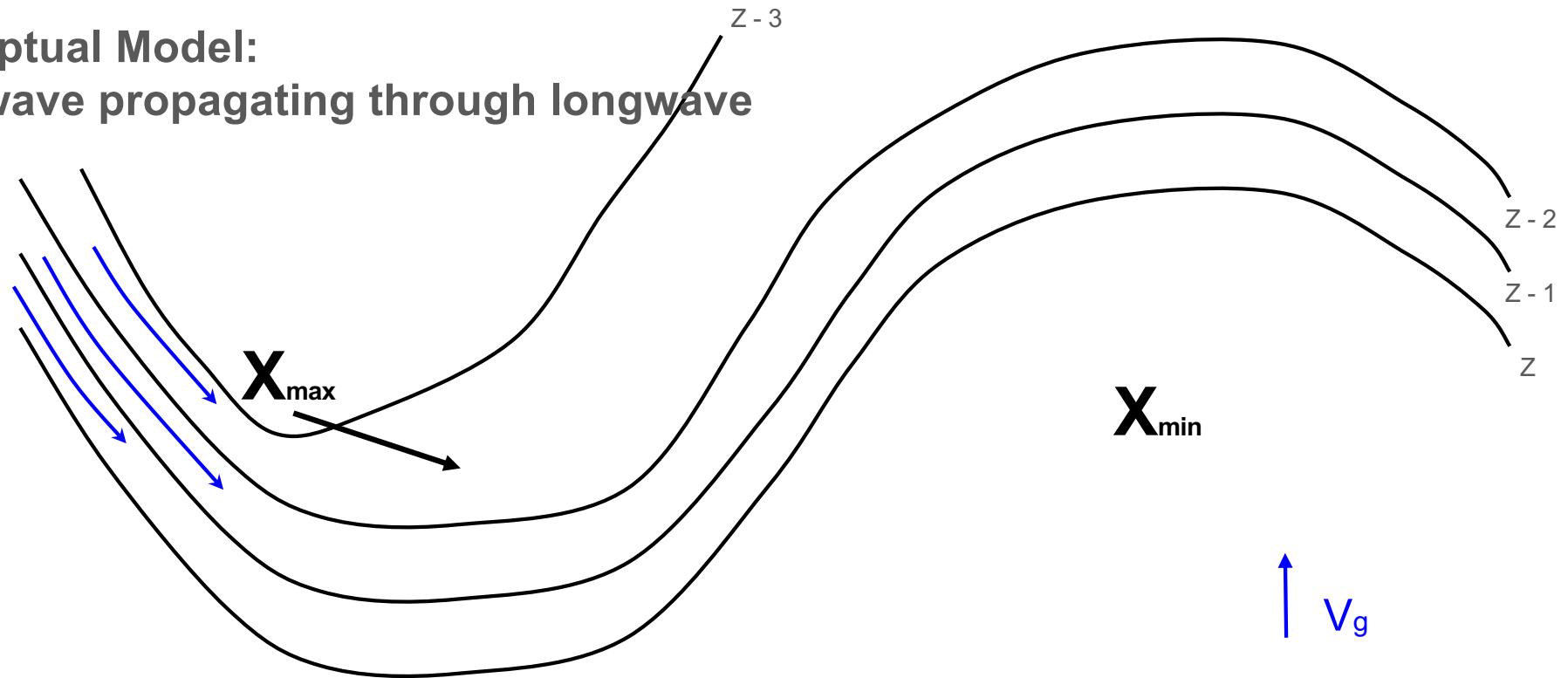
## How do we use this?

- Use the vorticity advection term to help anticipate where a vorticity maximum or minimum (i.e. a trough or a ridge) will go.
- Use the differential thermal advection term to anticipate whether or not a trough or ridge will amplify.
- The diabatic heating term is similar to the thermal advection term, but is typically not as consequential.

# QG X Application

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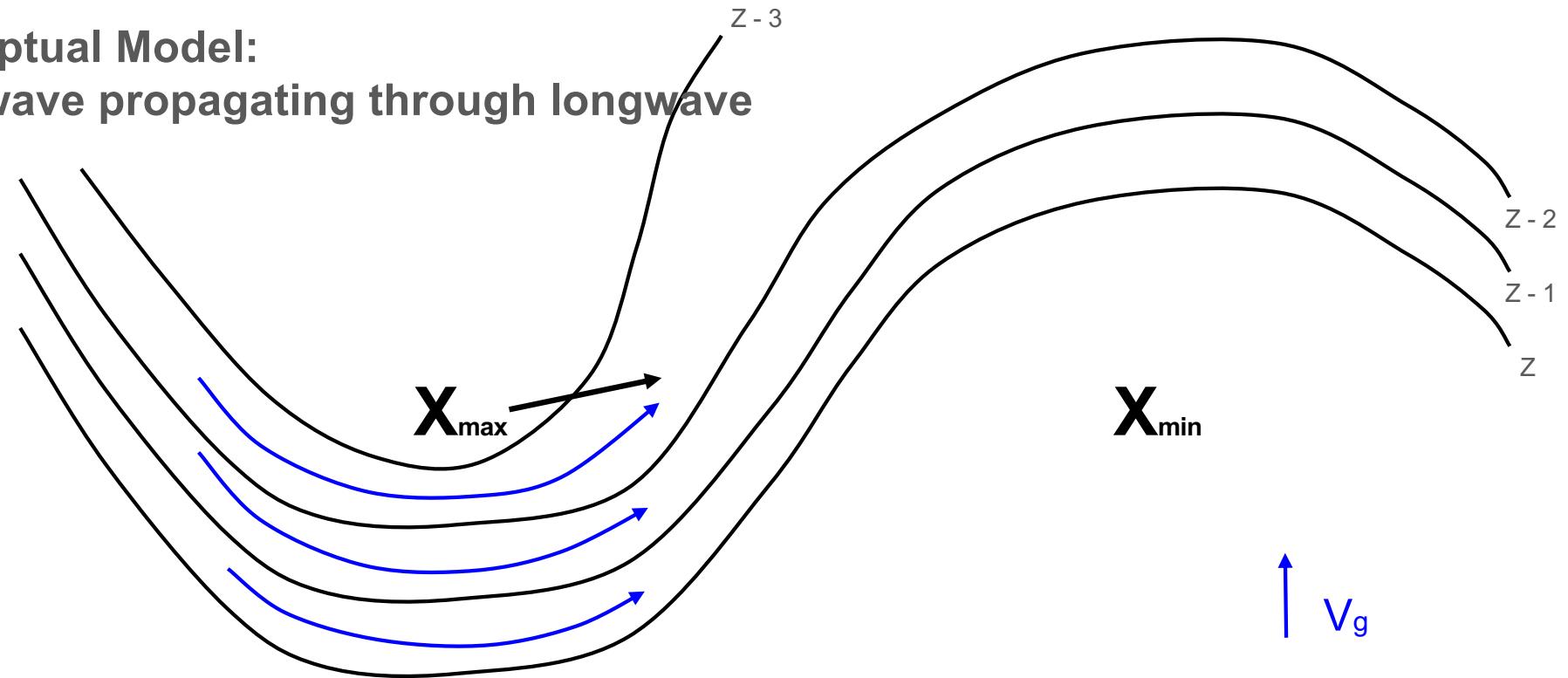
Conceptual Model:  
Shortwave propagating through longwave



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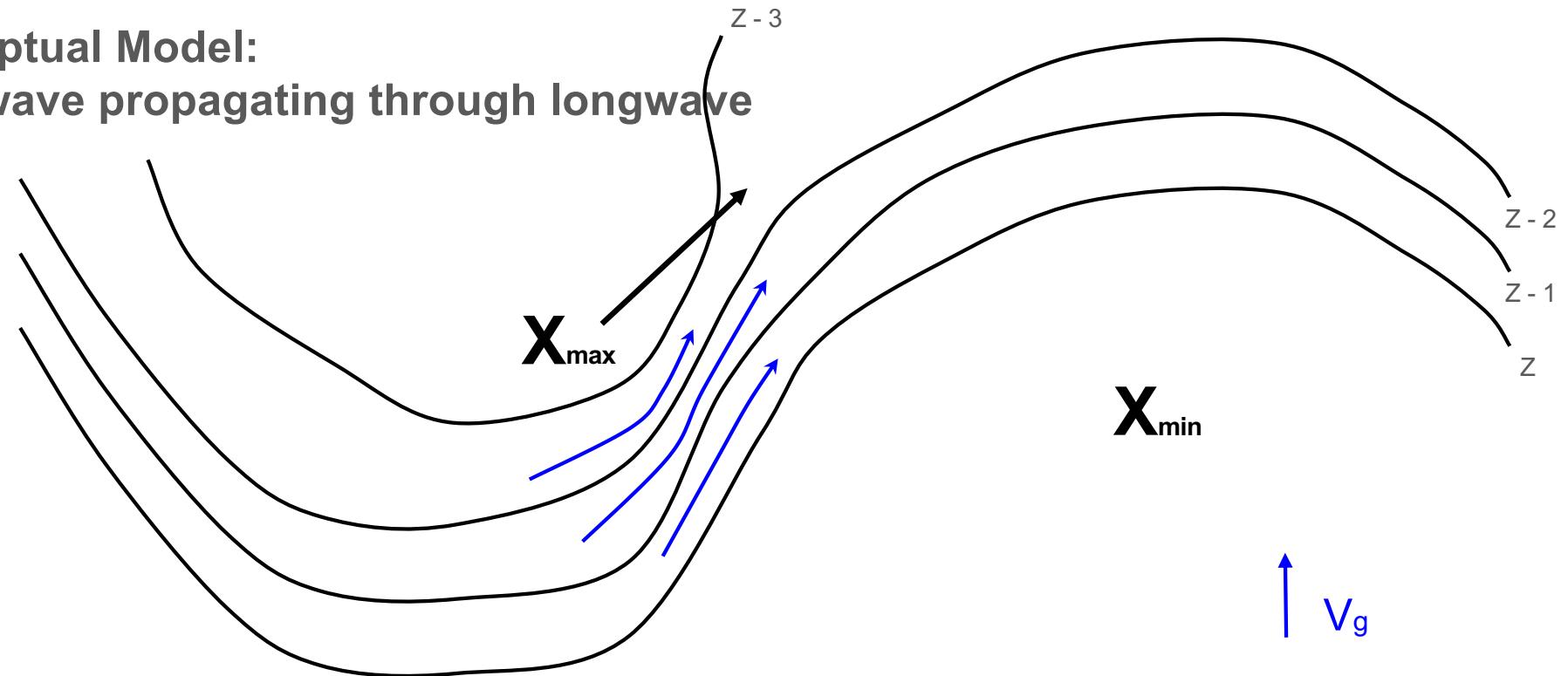
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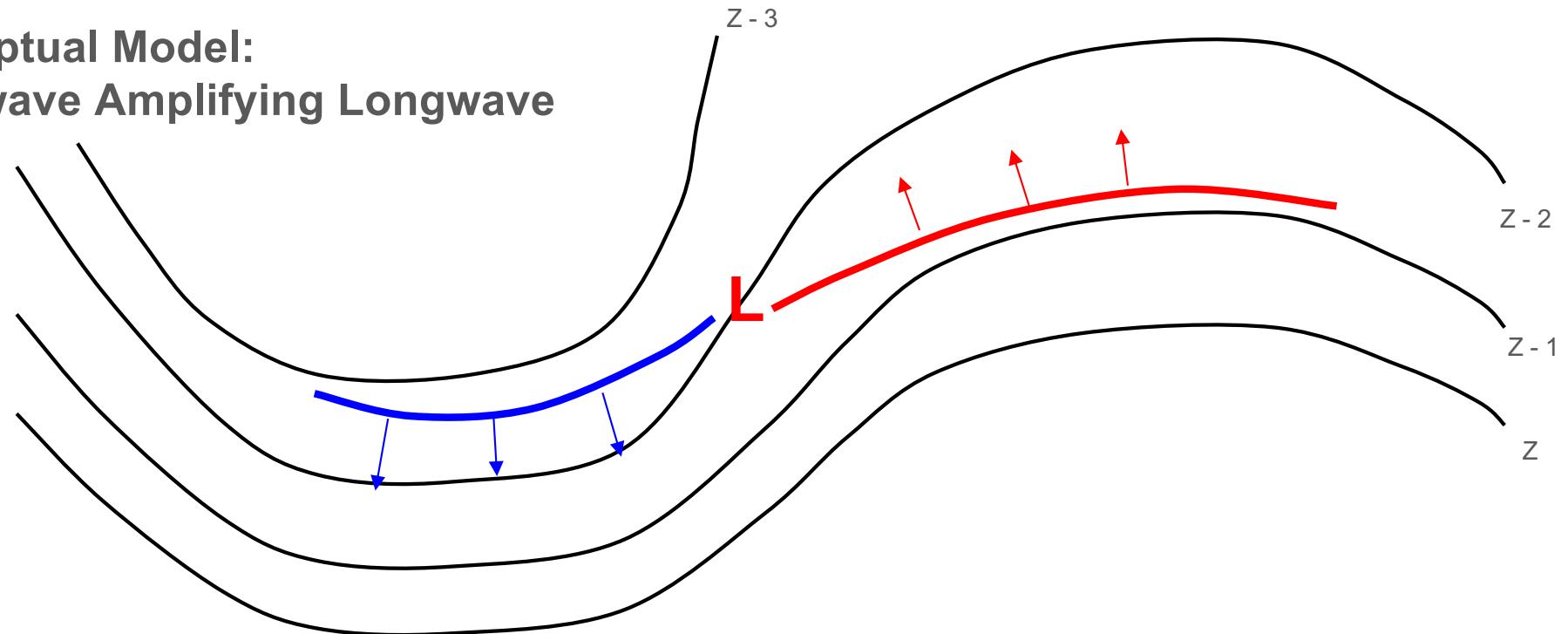
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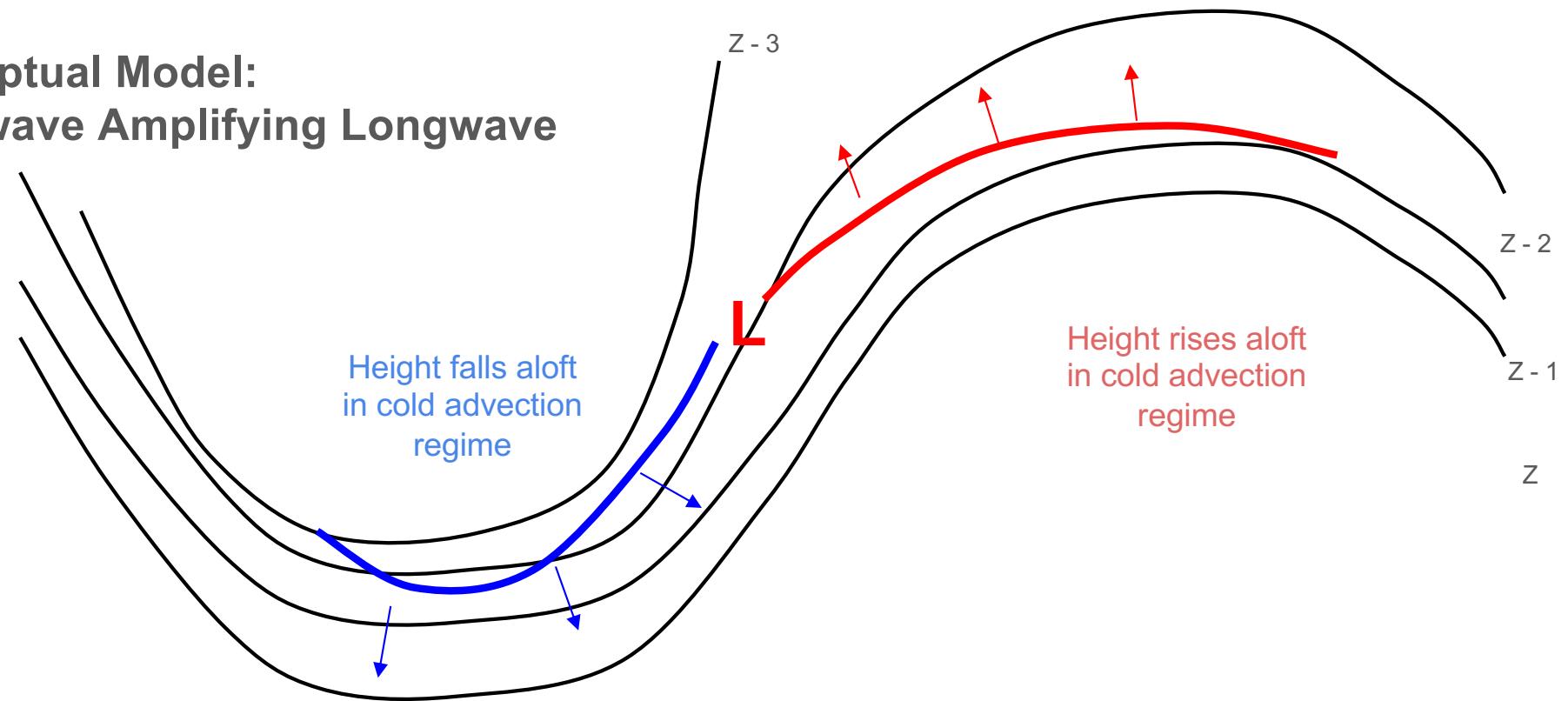
Conceptual Model:  
Shortwave Amplifying Longwave



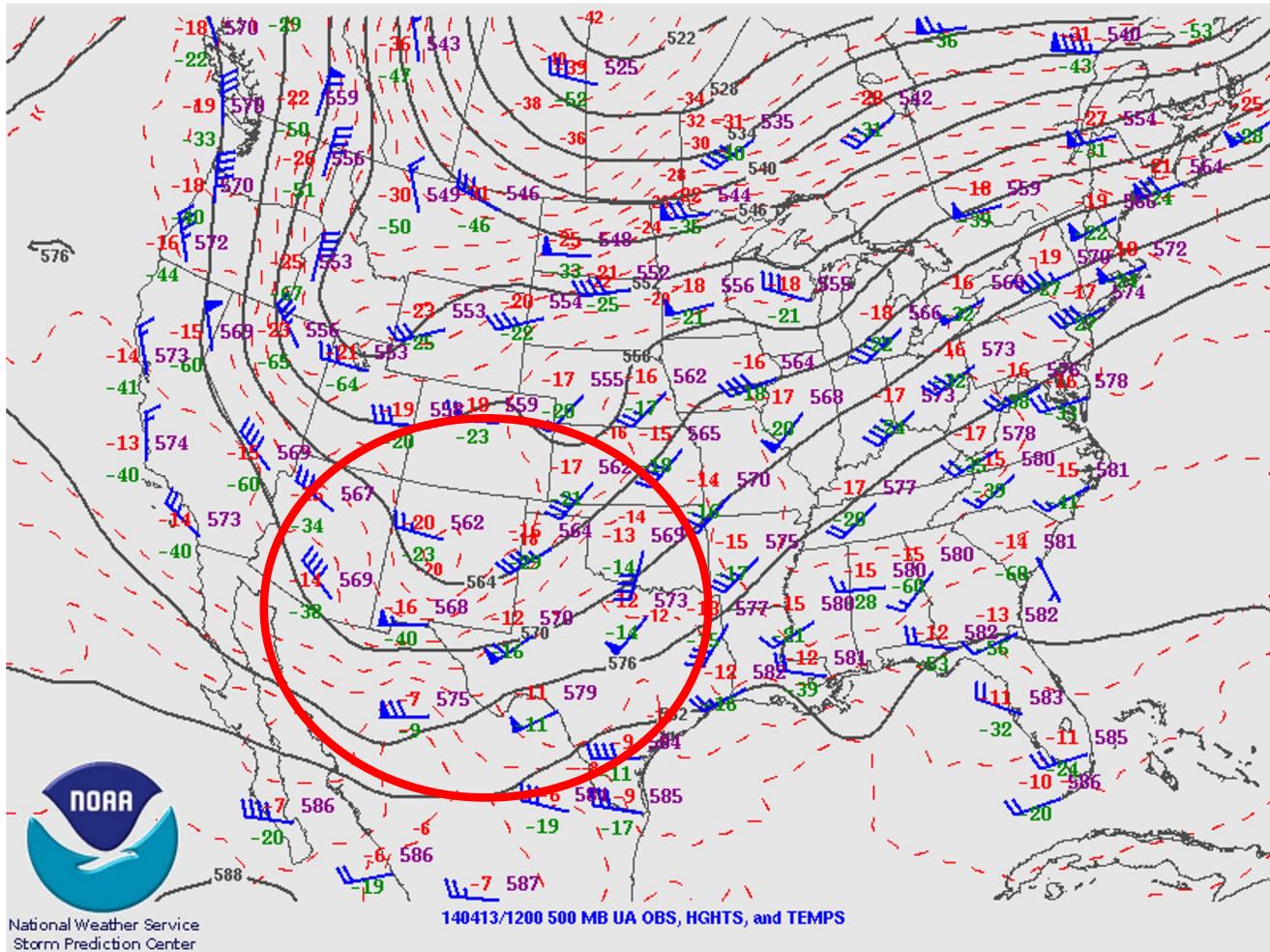
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Conceptual Model:  
Shortwave Amplifying Longwave



# QG X Examples

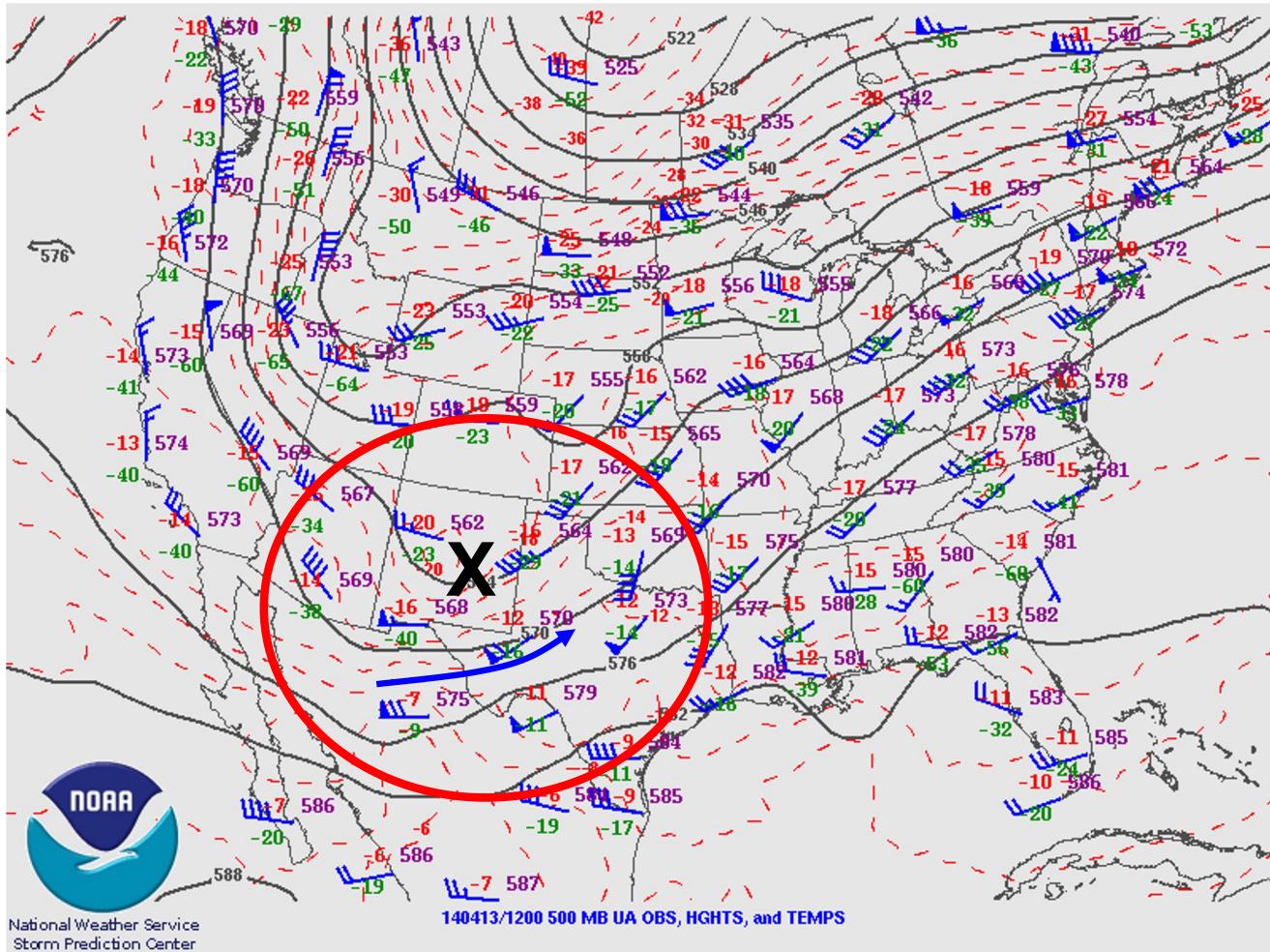


Focus on the shortwave trough in New Mexico.

Where is the vorticity maximum at?

Where will the 500 mb winds advect this vort. max?

# QG X Examples

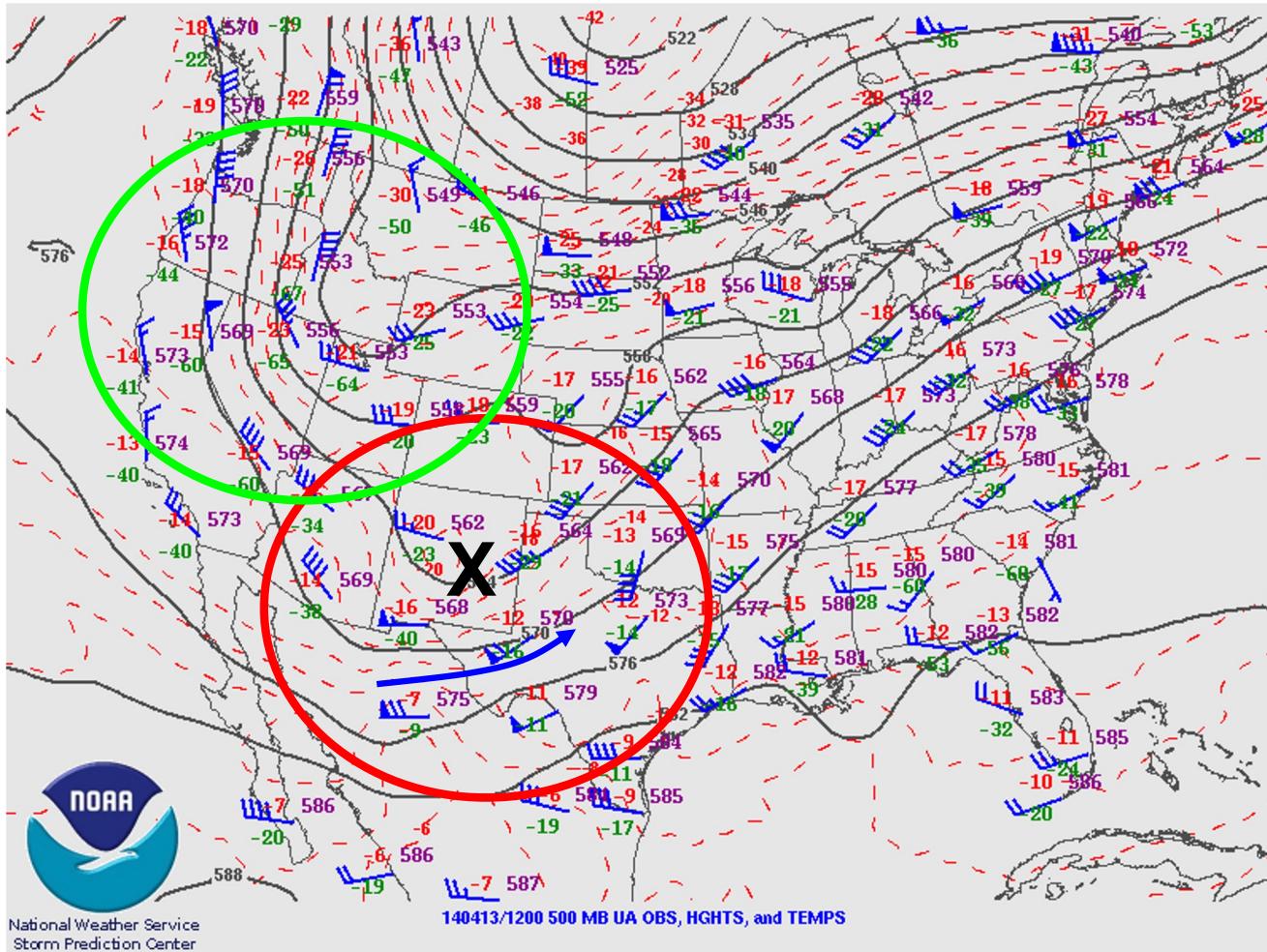


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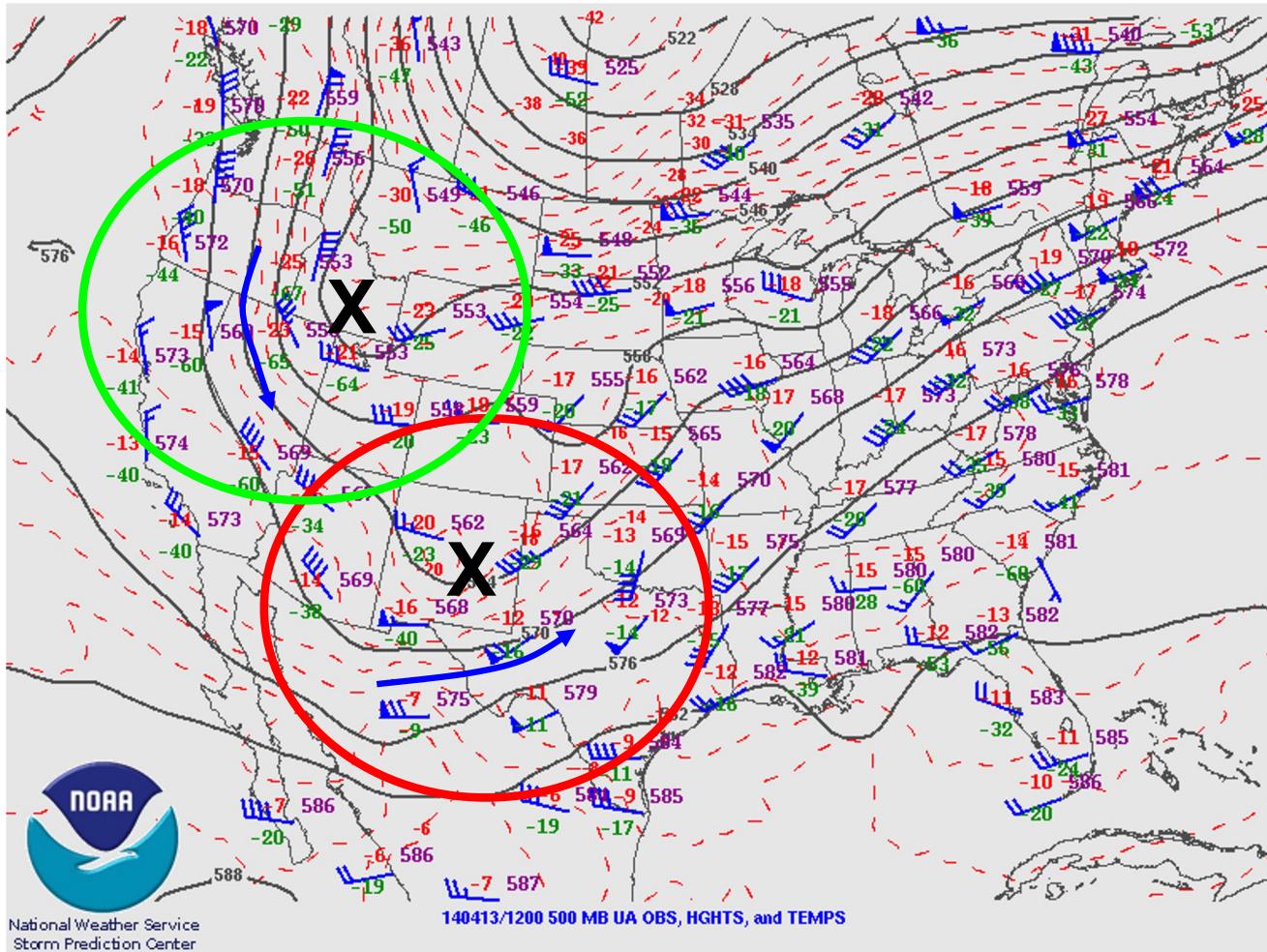


How about this shortwave over the northern Great Basin?

Where is the vorticity maximum?

Where will the geostrophic winds advect the vorticity?

# QG X Examples

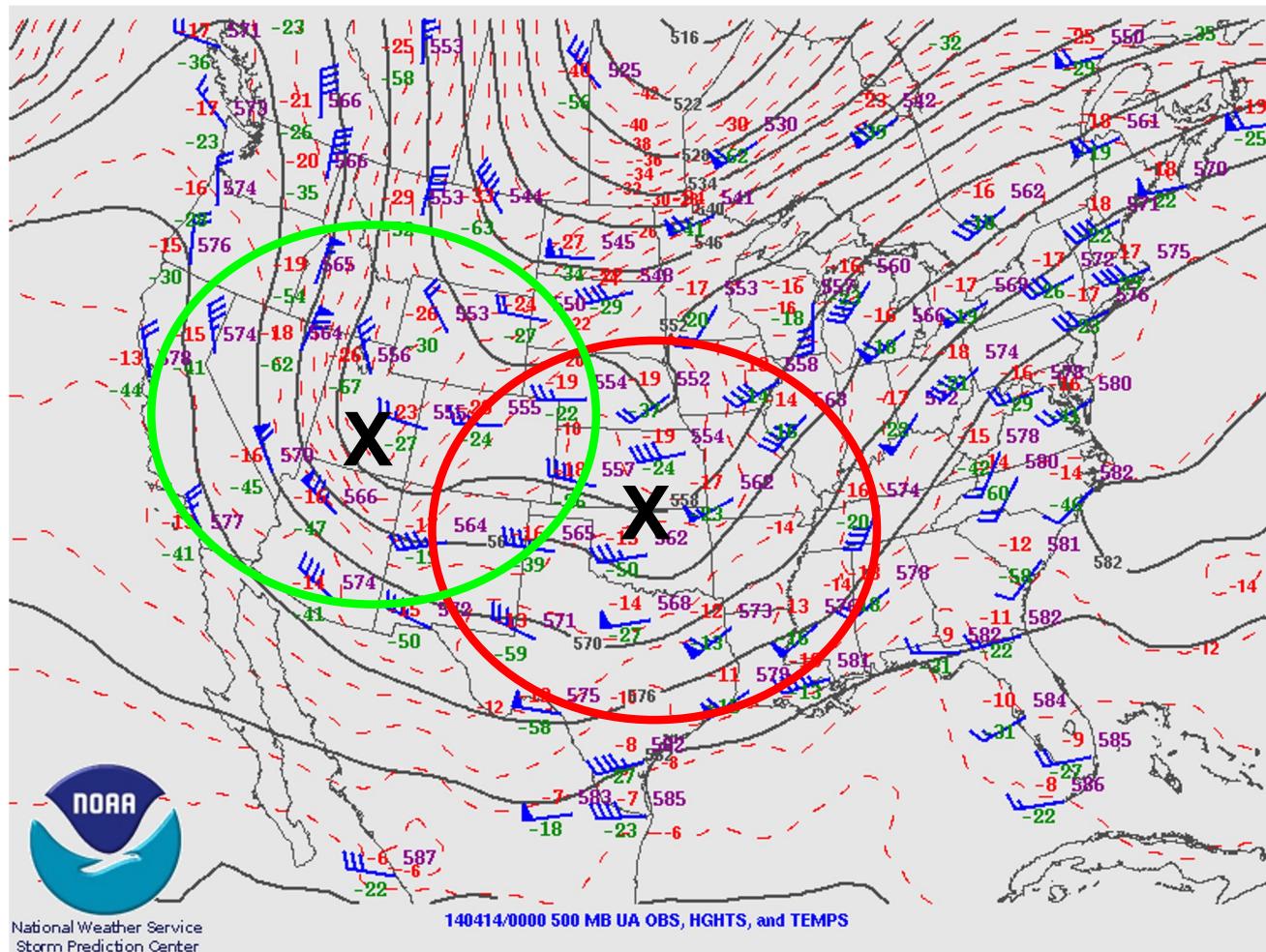


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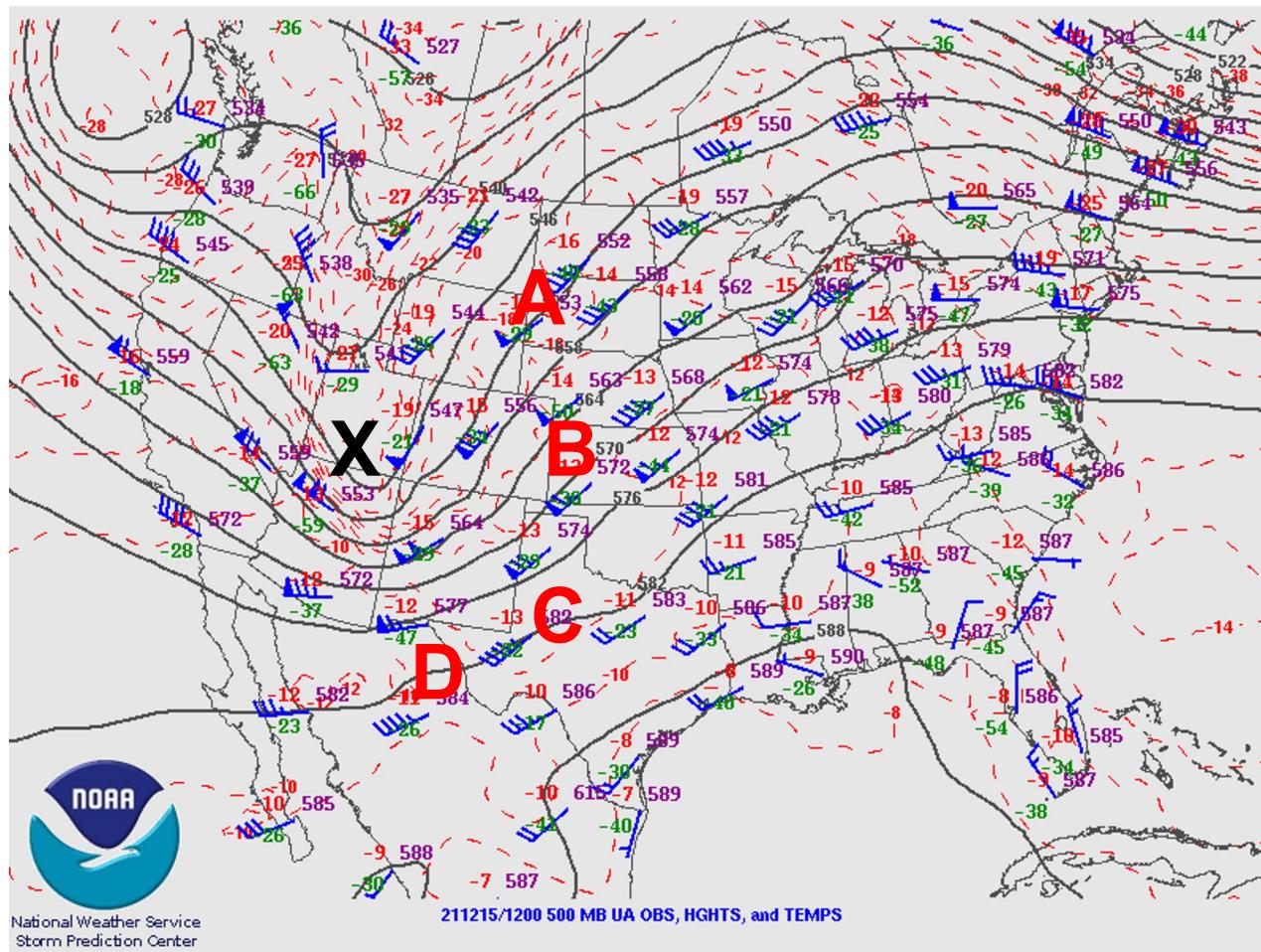
Where will the geostrophic winds advect the vorticity?

# QG X Examples



Did this meet your expectations?

# QG X Examples

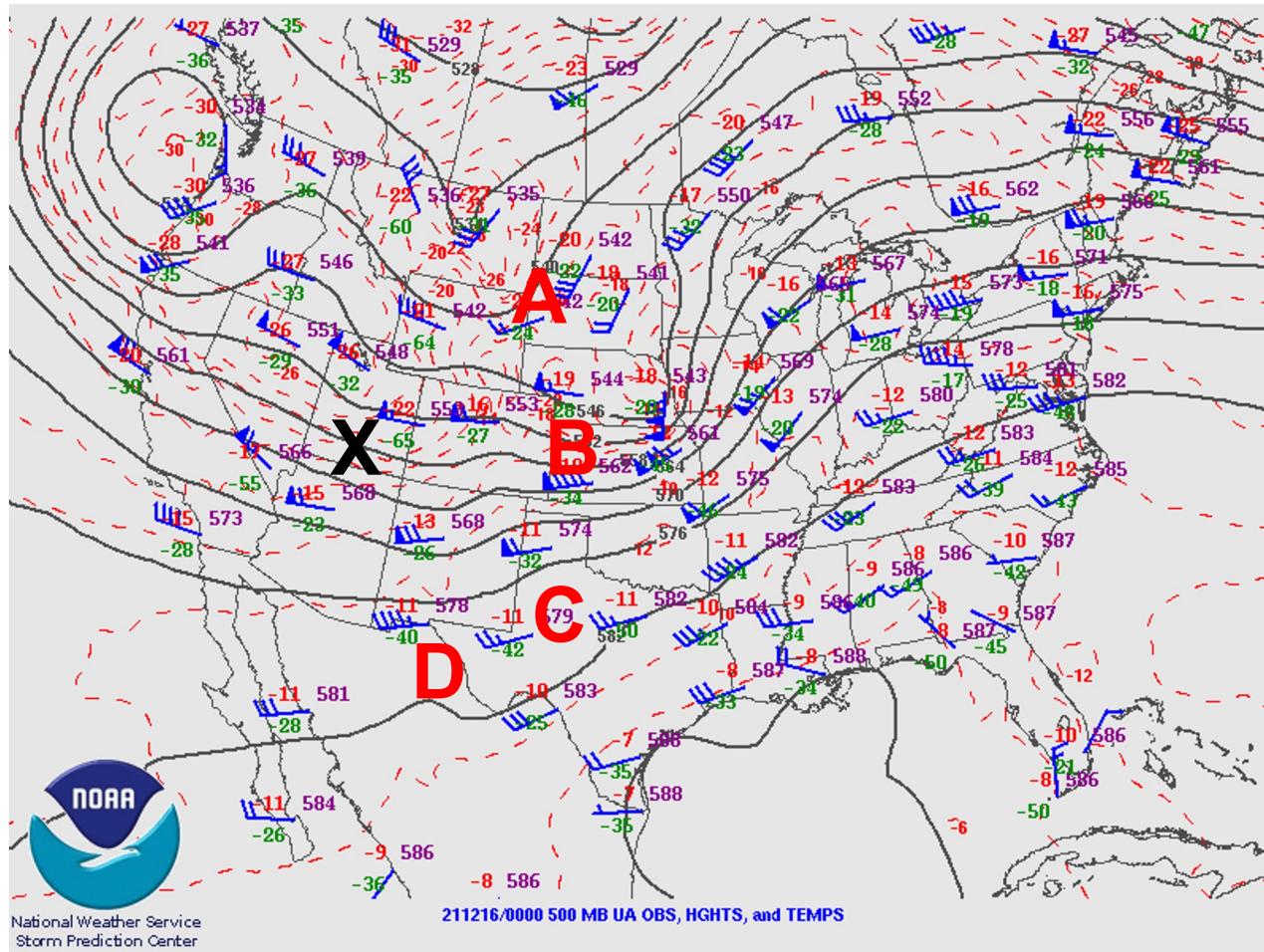


How about this case?

Where do you think this trough will go in the next 12 hours?

(Choose A, B, C, or D)

# QG X Examples



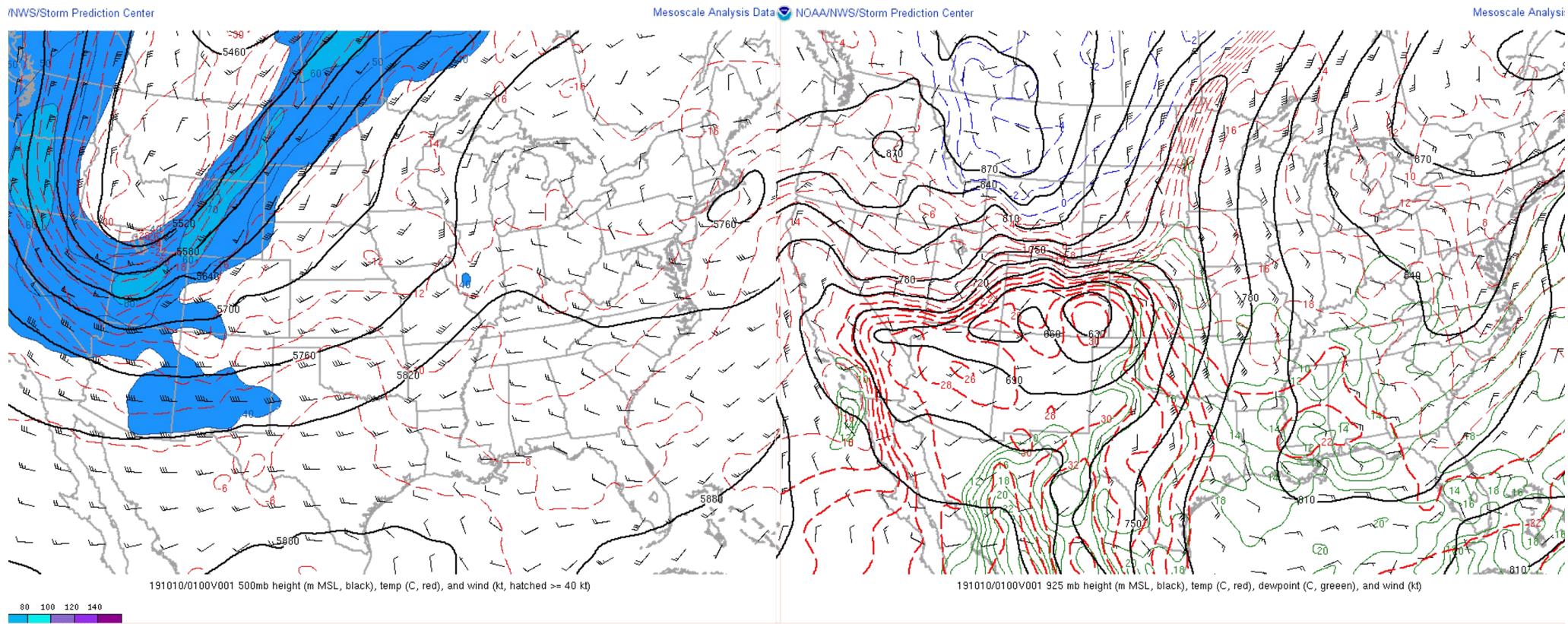
How about this case?

Where do you think this trough will go in the next 12 hours?

(Choose A, B, C, or D)

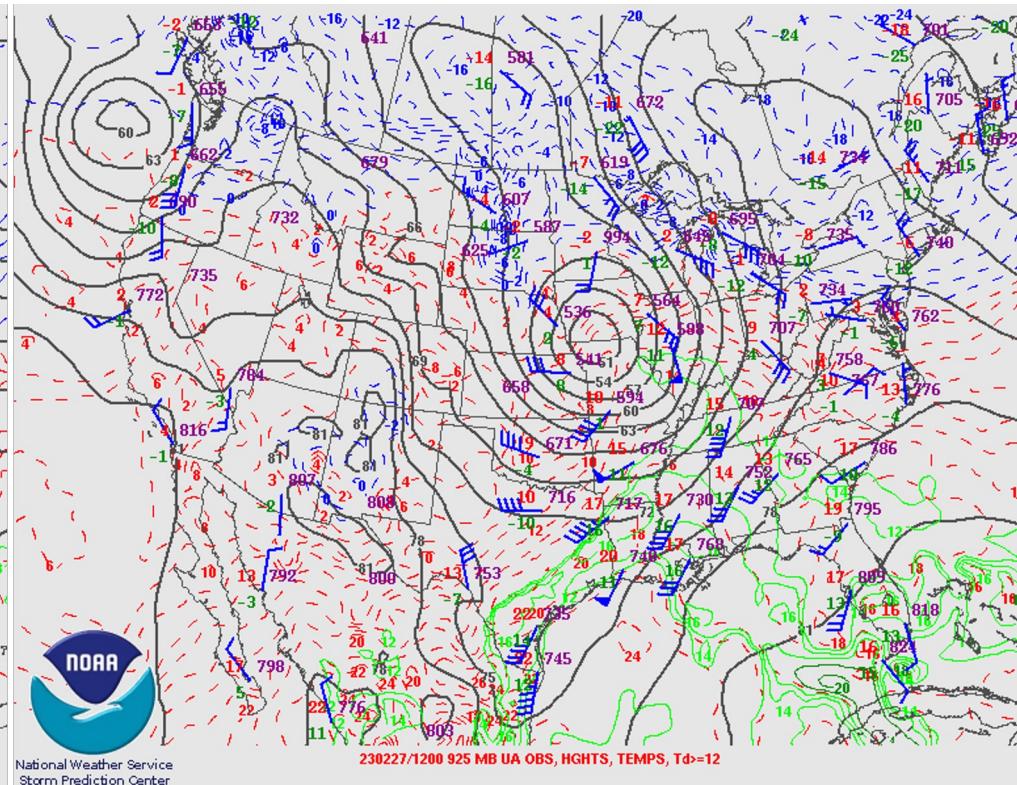
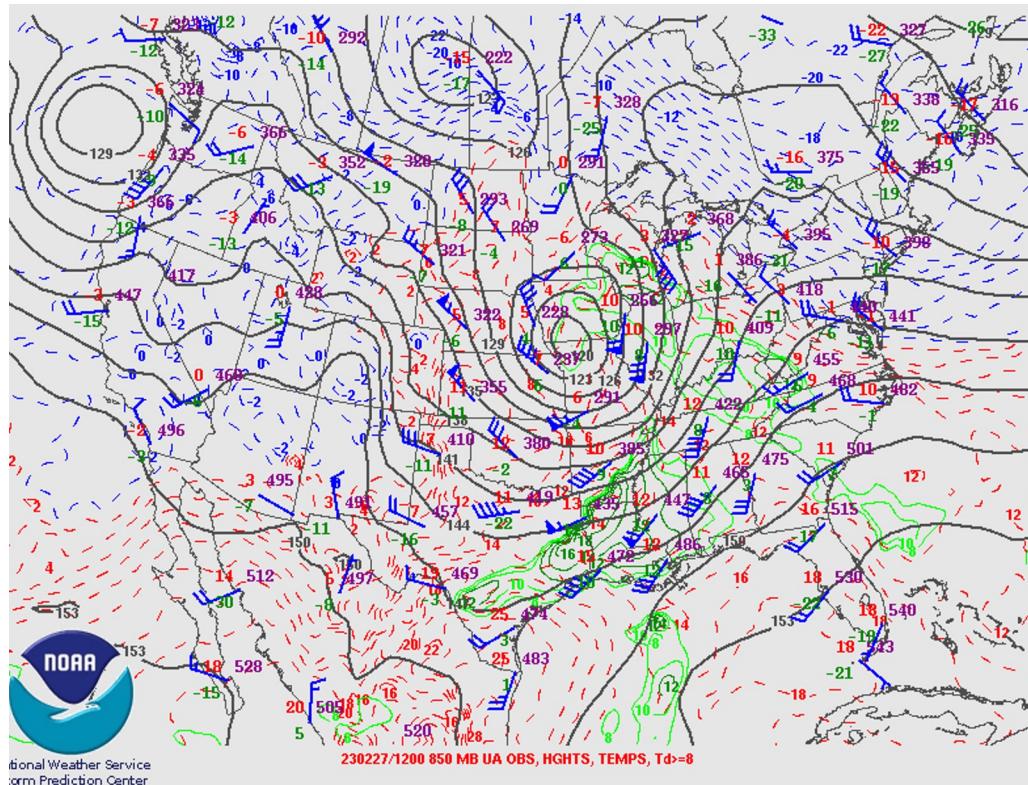
# QG X Examples

Watch how the 500 mb trough deepens as the 925 mb cold front surges south into the Plains.  
This is an example of differential thermal advection.



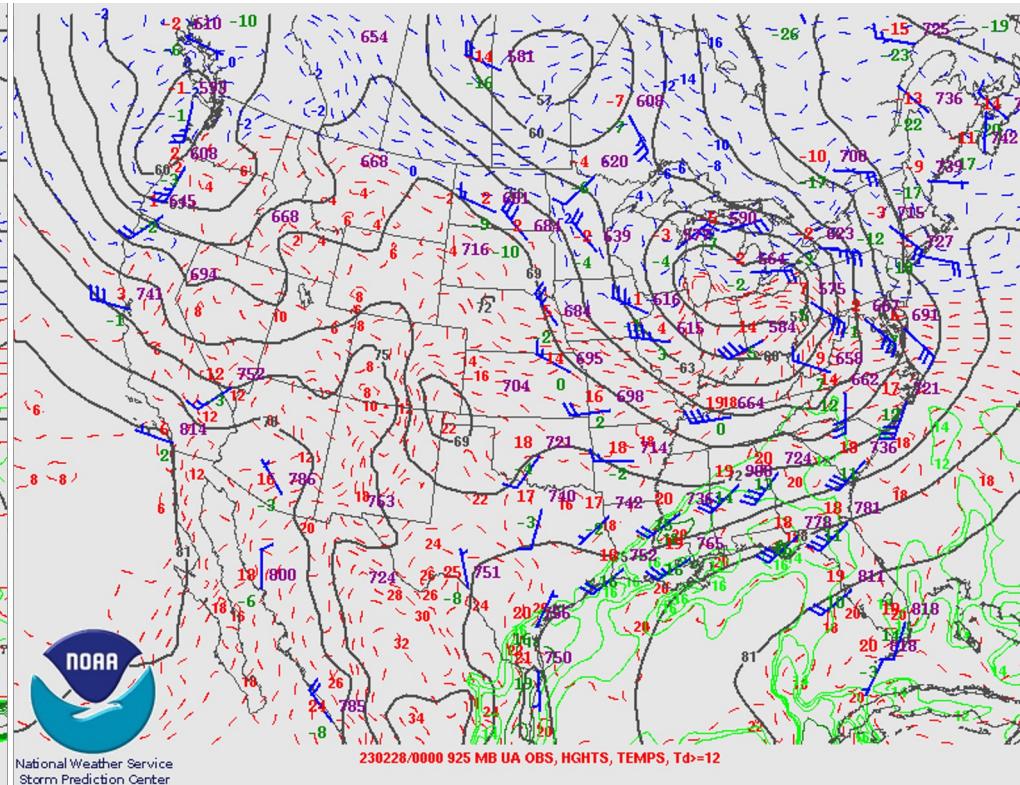
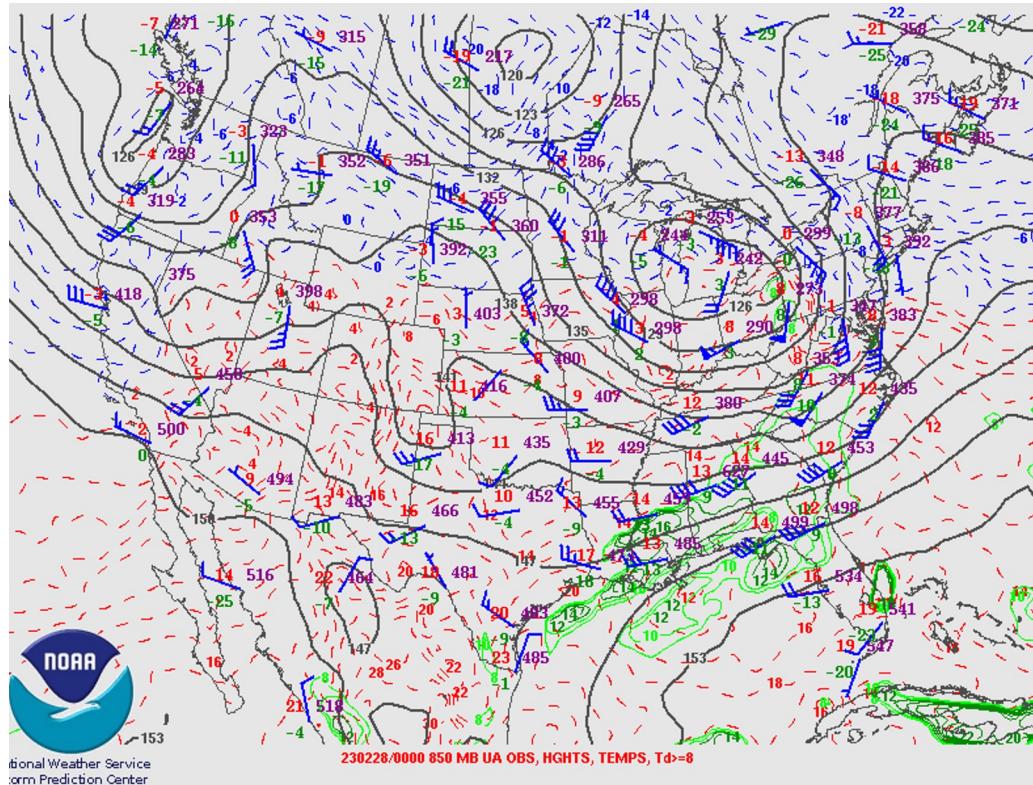
# QG X Examples

Where is the strongest warm air advection at 850 mb?  
This will tell you where the 925 mb low should go!



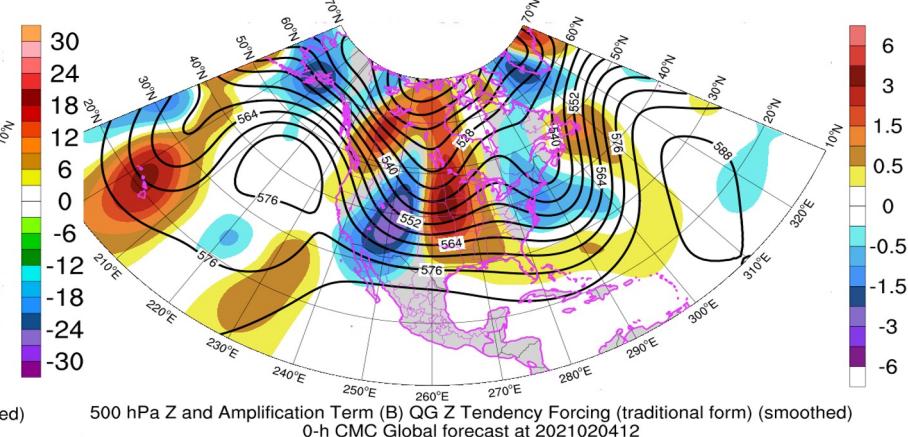
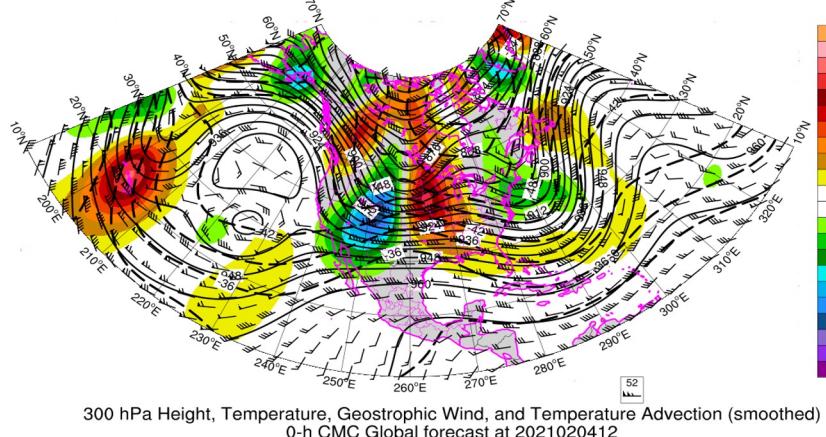
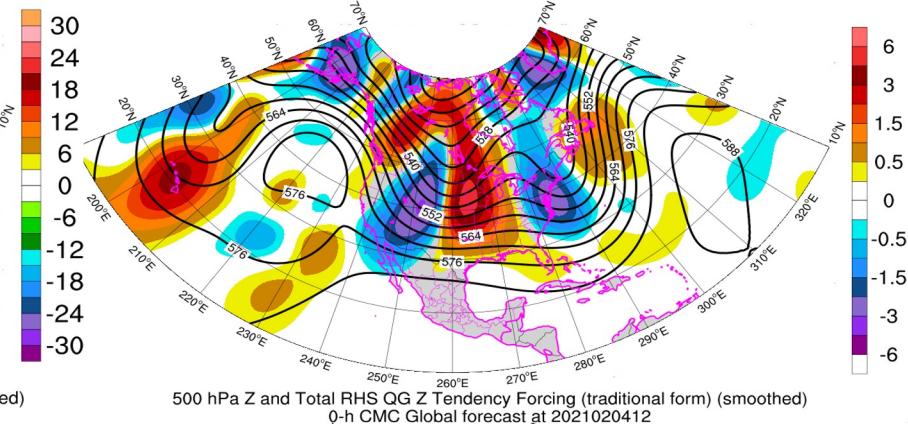
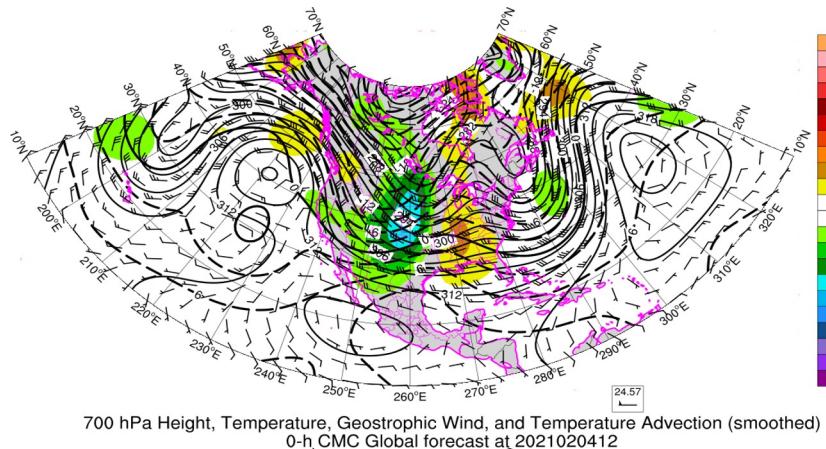
# QG X Examples

Where is the strongest warm air advection at 850 mb?  
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# QG Resources

<https://inside.nssl.noaa.gov/tgalarneau/real-time-qg-diagnostics/>

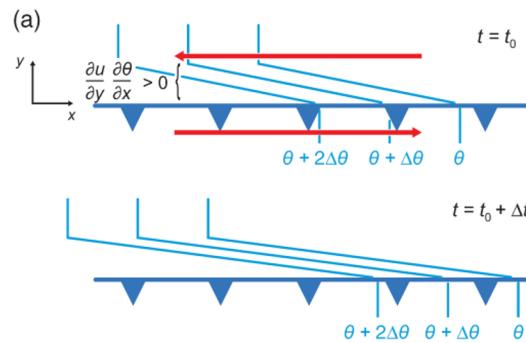


# Frontogenesis

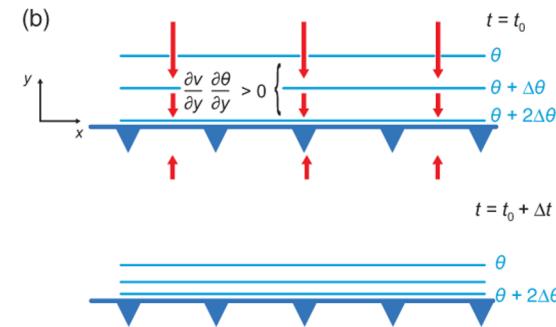
- Frontogenesis (frontolysis) is the strengthening (weakening) of a temperature gradient
- In case of frontogenesis, thermal wind balance (TWB) is violated because temperature gradient is too strong for the given wind shear
  - To restore TWB, atmosphere weakens temperature gradient via ascent (adiabatic cooling) on warm side and descent (adiabatic warming) on cold side of gradient
- Fronts are zones where thermal advection and frontogenesis are easily enhanced and are also preferred corridors for cyclones/cyclogenesis

# Frontogenesis Mechanisms

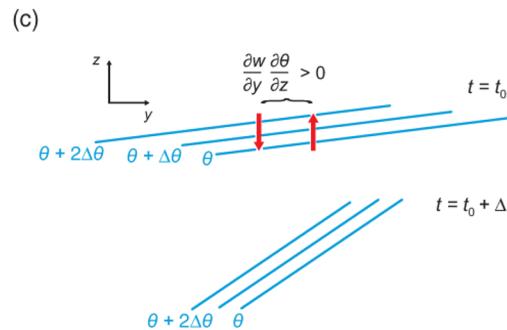
horizontal shear



confluence



tilting



Differential diabatic heating

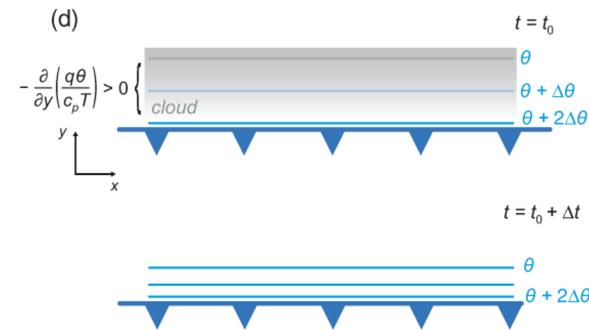


Fig. 5.4 MR

# Frontogenesis and Frontolysis by Confluence

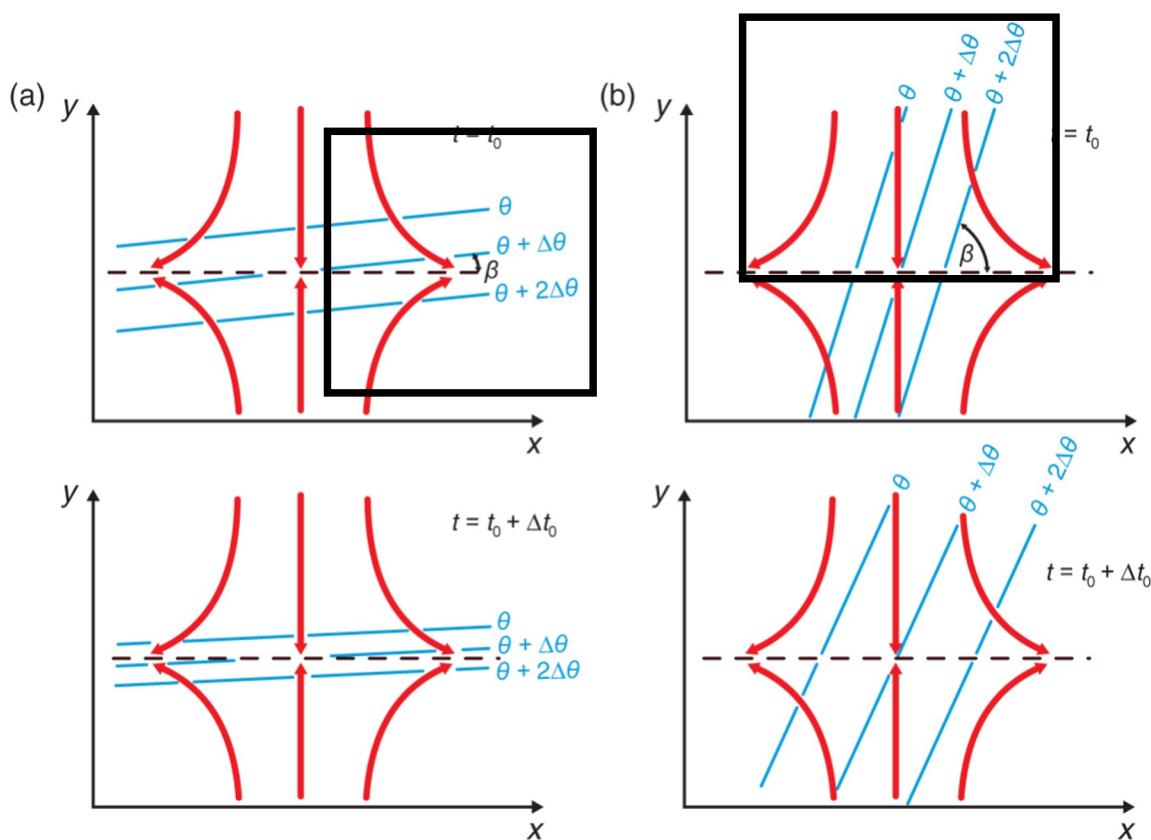
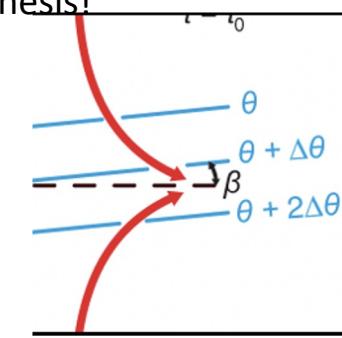
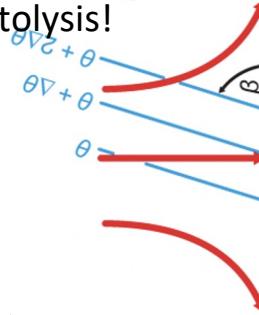


Fig. 5.5 MR  
2010

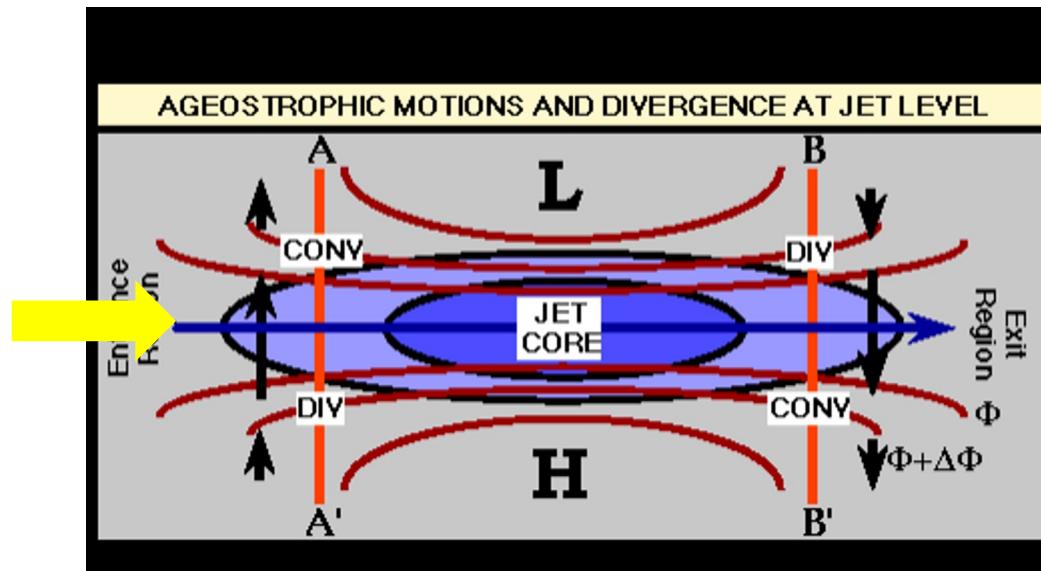
Like a jet entrance region:  
Frontogenesis!



Like a jet exit region:  
Frontolysis!



# Frontogenesis and Jet Streaks



Air entering jet streak – sinking on cold side, rising on warm side.

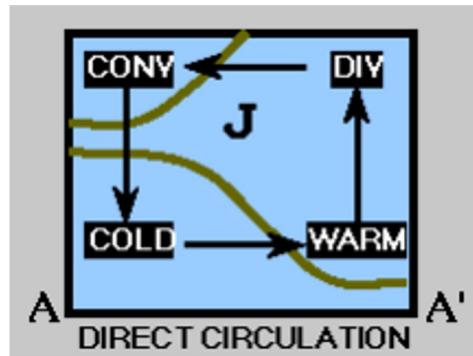
Air leaving jet streak – rising on cold side, sinking on warm side.

# Vertical Wind Shear

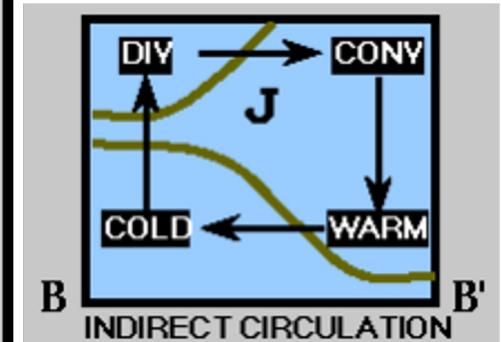
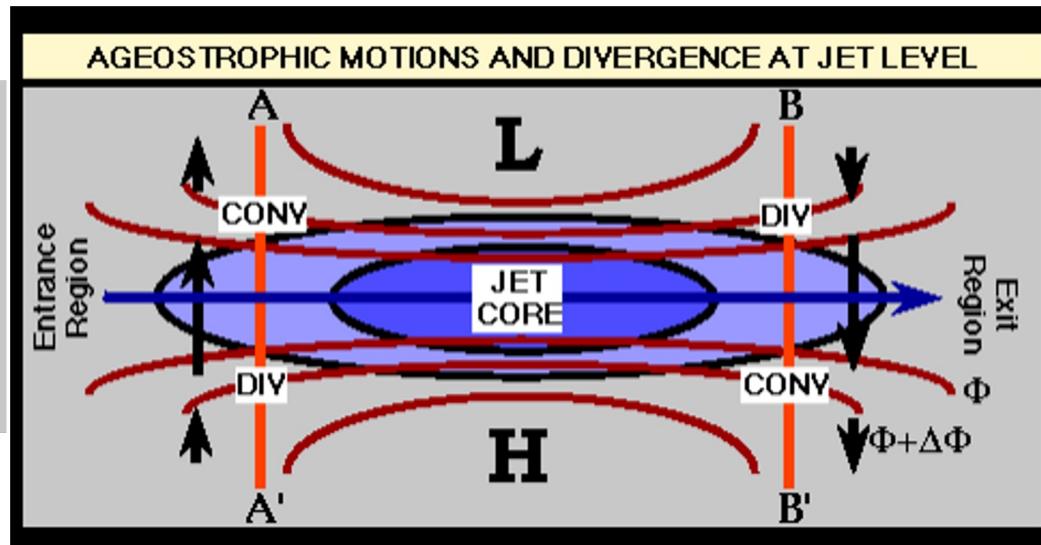
Where does it come from?

Secondary contributions:

Large accelerations of the horizontal wind due to large ageostrophic winds (think near jet streaks, areas of frontogenesis, and/or rapidly intensifying cyclones).



Erodes horizontal temperature gradient (weaker thermal wind)



Enhances horizontal temperature gradient (stronger thermal wind!)

For additional reading: M.R. 2010 and Doswell

# Frontogenesis and Jet Streaks

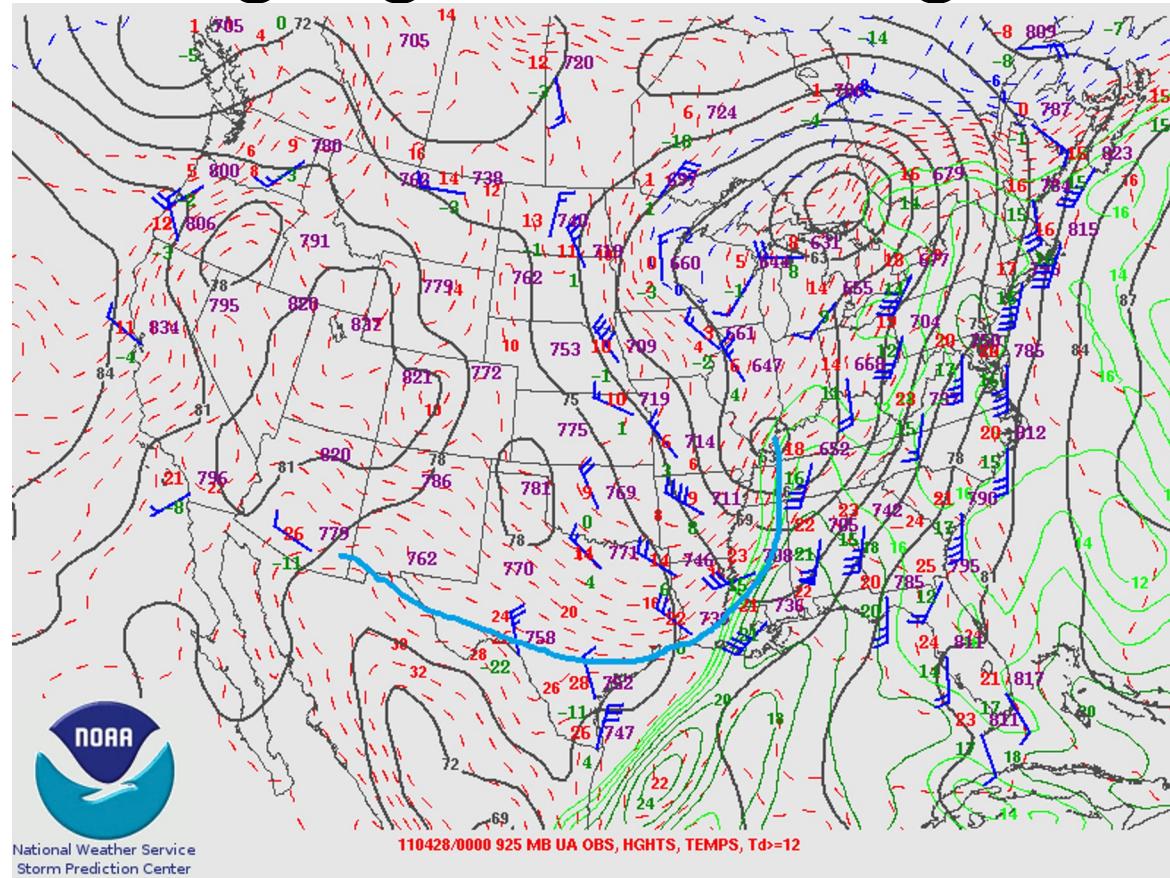
- Jet streaks are coincident with strong temperature gradients (thermal wind balance!)
- Air flows through a jet streak
  - Encounters strengthening temperature gradient (frontogenesis) in entrance region
  - Encounters weakening temperature gradient (frontolysis) in exit region
- Response to frontogenesis in entrance region is ascent on warm side (right entrance) and descent on cold side (left entrance)
- Response to frontolysis in exit region is?

# Baroclinic systems

- Vorticity and thermal structure tilts westward (upstream) with height
  - Deepening/strengthening systems
  - Differential thermal advection leads to destabilization
- Warm advection corresponds to veering winds with height
  - Large clockwise turning hodographs in warm sector
- Strong jet streaks and fronts are present

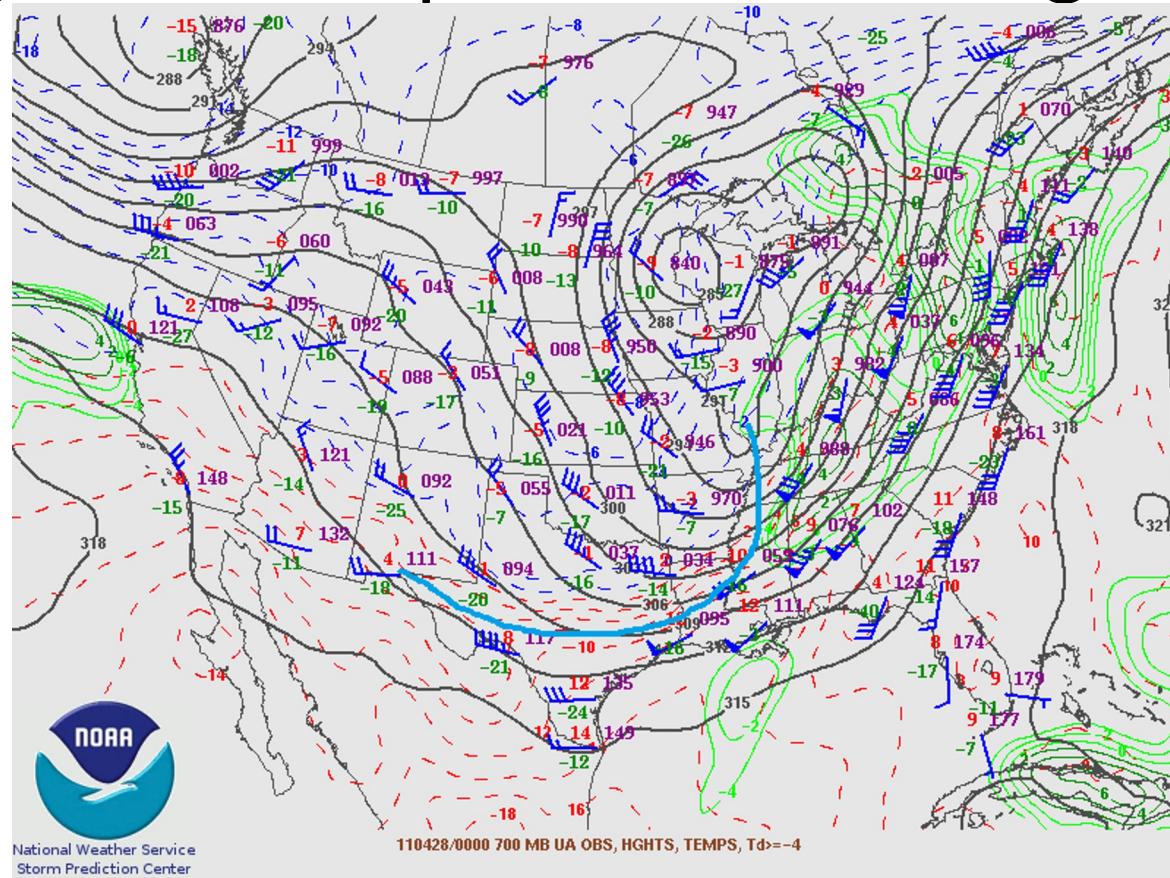
# Edge of stronger gradient near ground

925m  
b

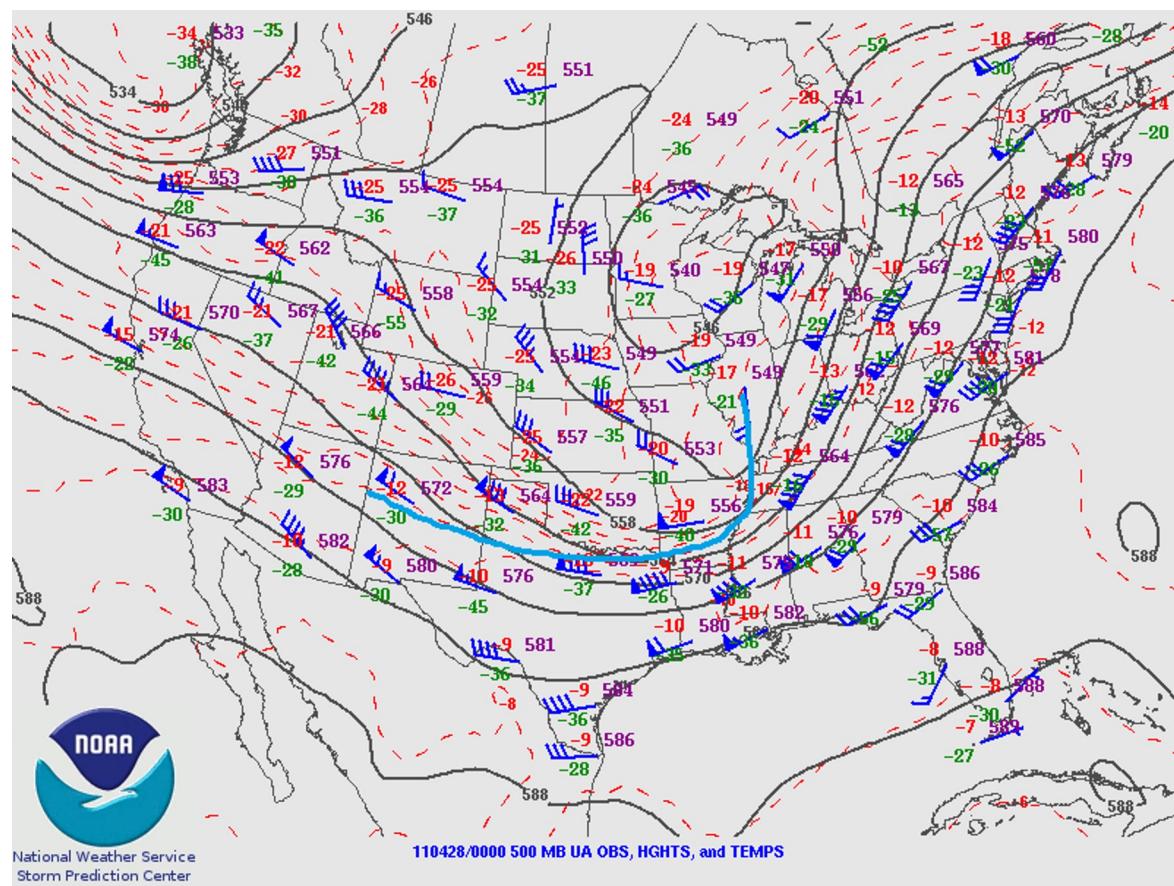


# Temp gradient slopes NW with height

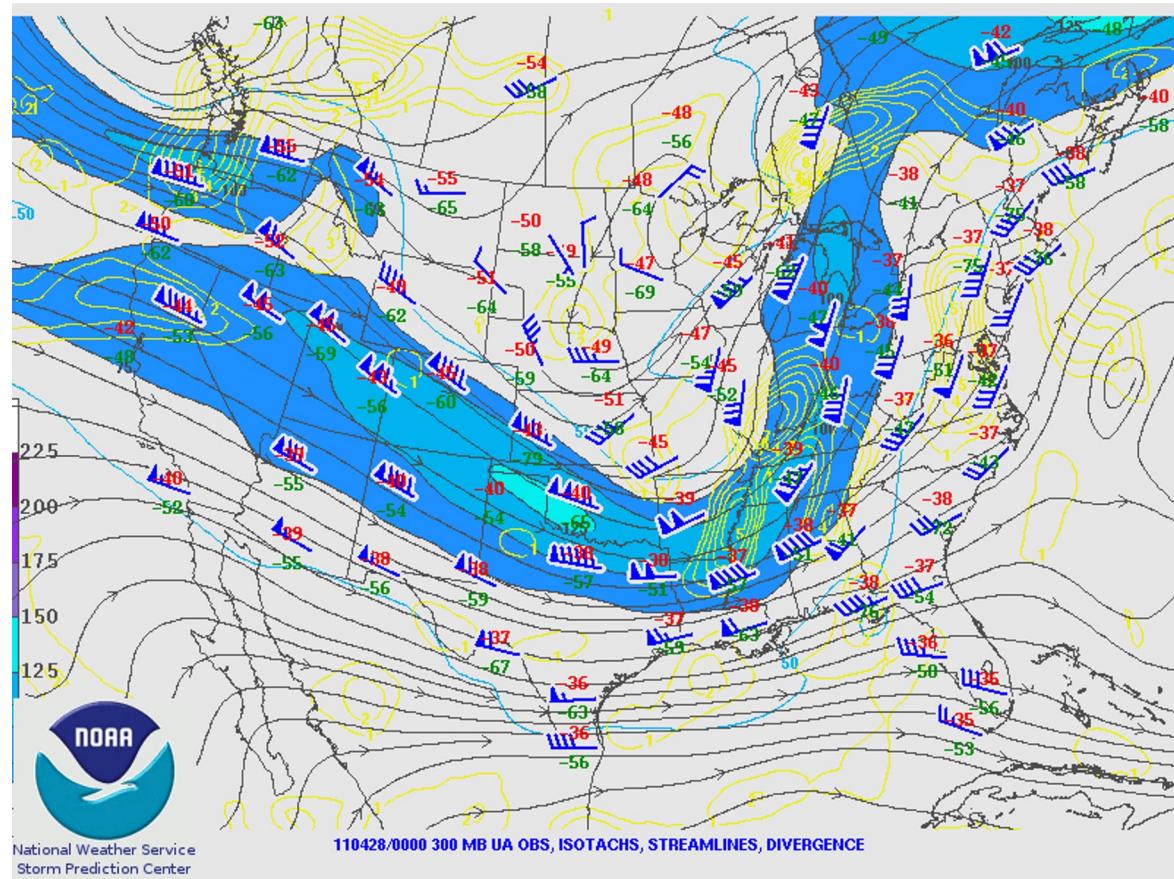
700m  
b



500m  
b

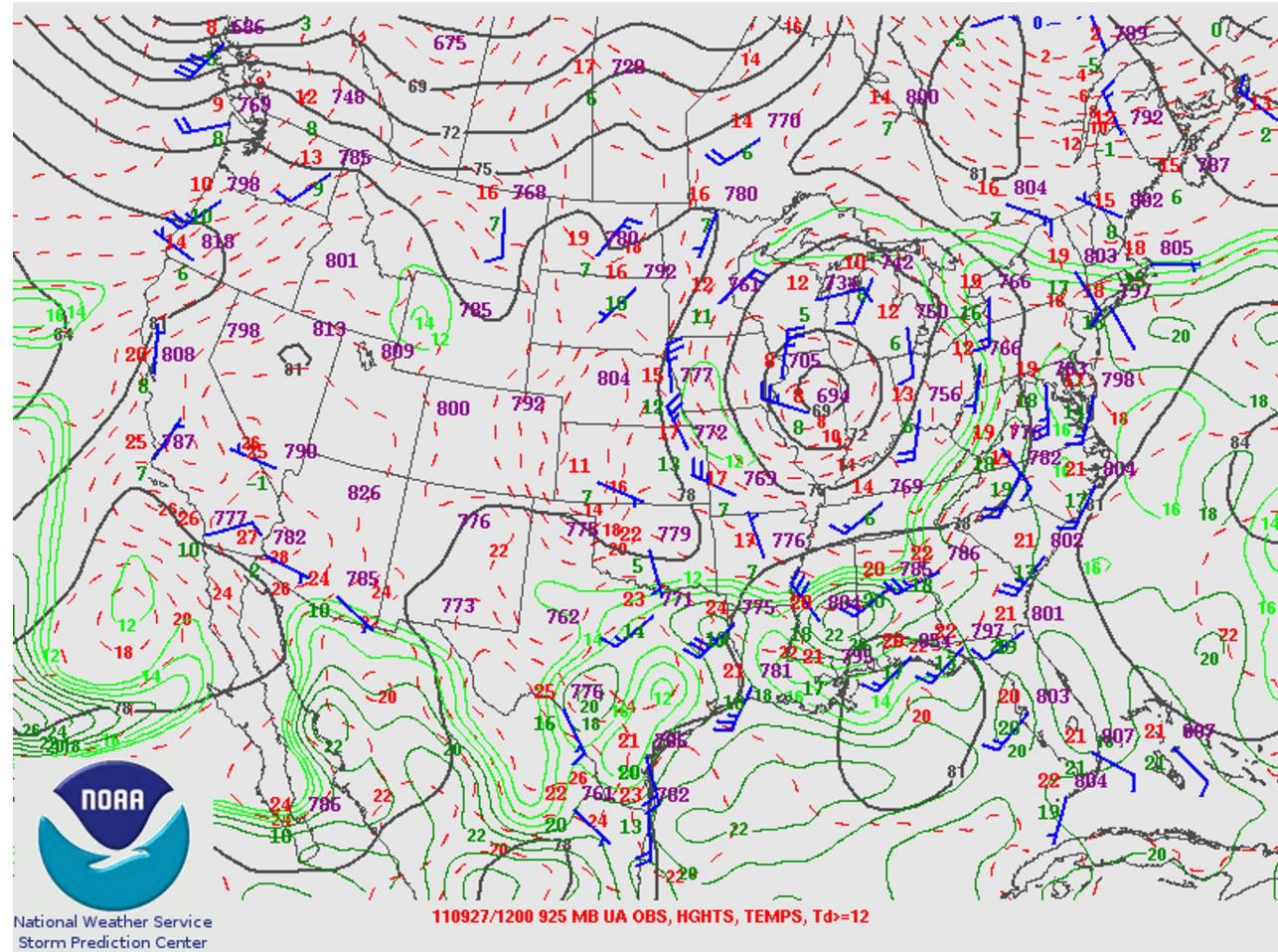


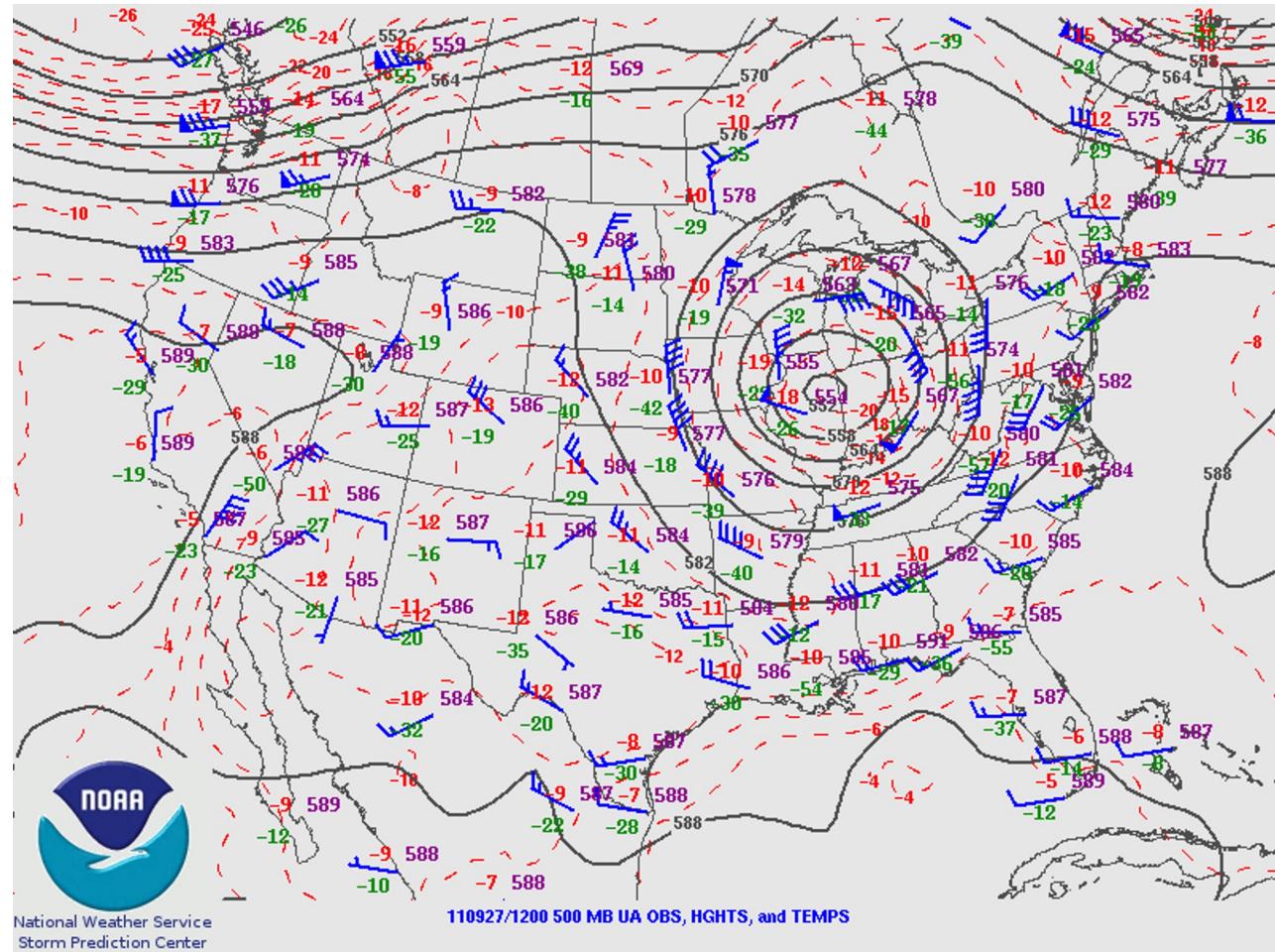
300m  
b



# Equivalent Barotropic Systems

- Vorticity and thermal structure vertically stacked with height
  - Steady state or weakening systems
- Minimal temperature advection corresponds to weak vertical shear
  - No feedback mechanism to intensify.
- Weak gradients – weak flow – slow moving
  - Can even retrograde if large enough because planetary vorticity begins to dominate the relative component.





## Final Comments

- Can explain development of weather systems (QG height tendency and QG cyclogenesis)
- Can explain why ascent occurs where it does near troughs, jets, and fronts
- Look for gradients!
- Synoptic-scale processes set the stage for severe thunderstorm development