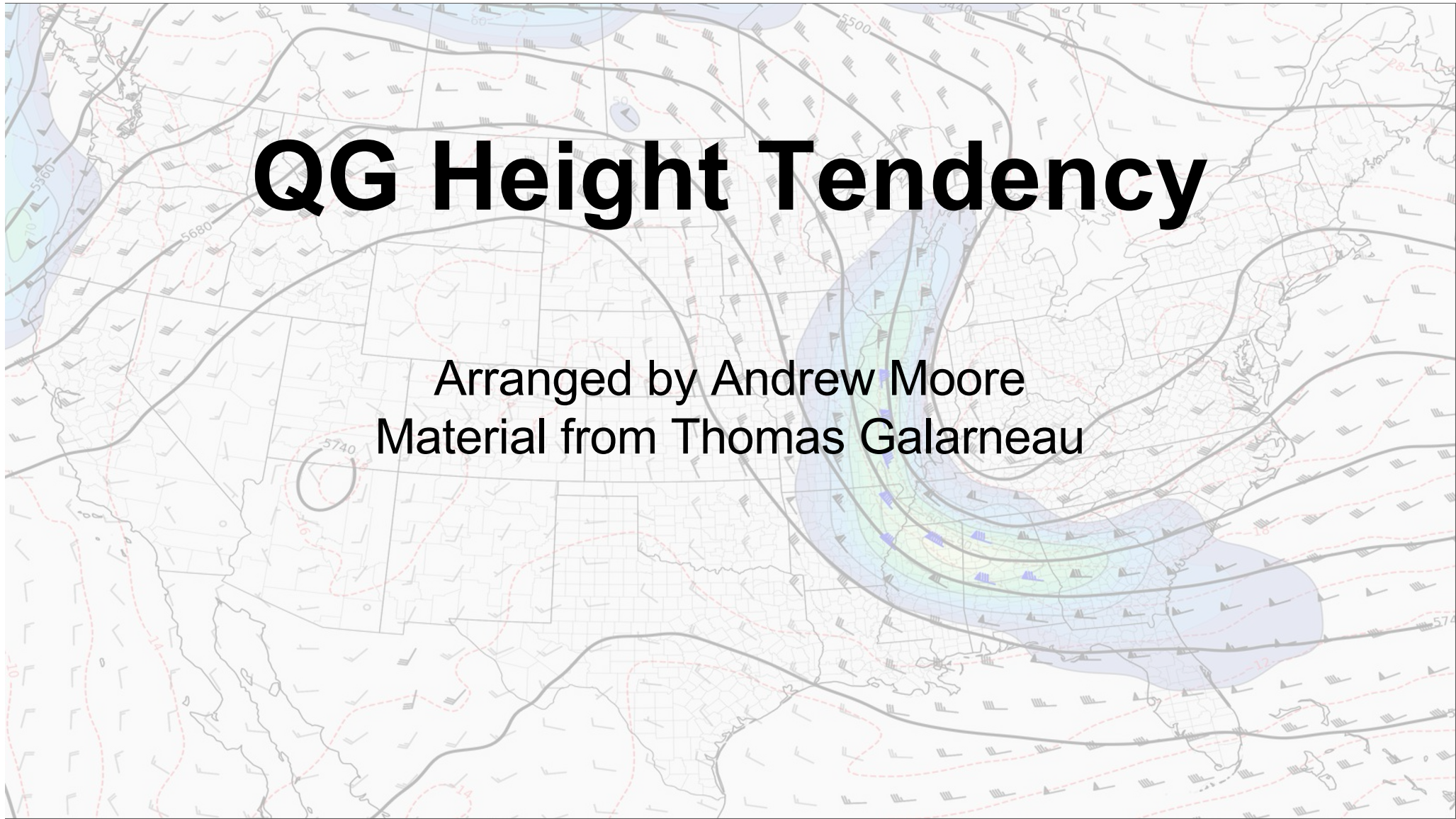


QG Height Tendency

Arranged by Andrew Moore
Material from Thomas Galarneau



QG Height Tendency

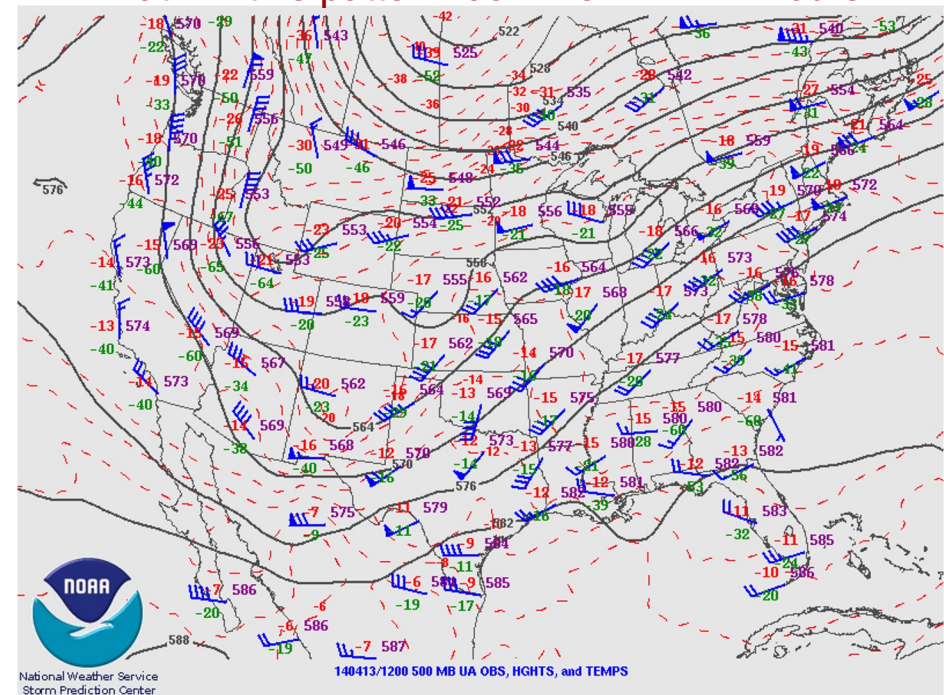
Why do we care?

The QG height tendency equation allows us to anticipate:

- The evolution of upper air and surface patterns
- The evolution of certain severe weather parameters (e.g. shear, lift, etc...)

It is also relatively easy to use!

What will this pattern look like in 12-24 hours?



QG Height Tendency

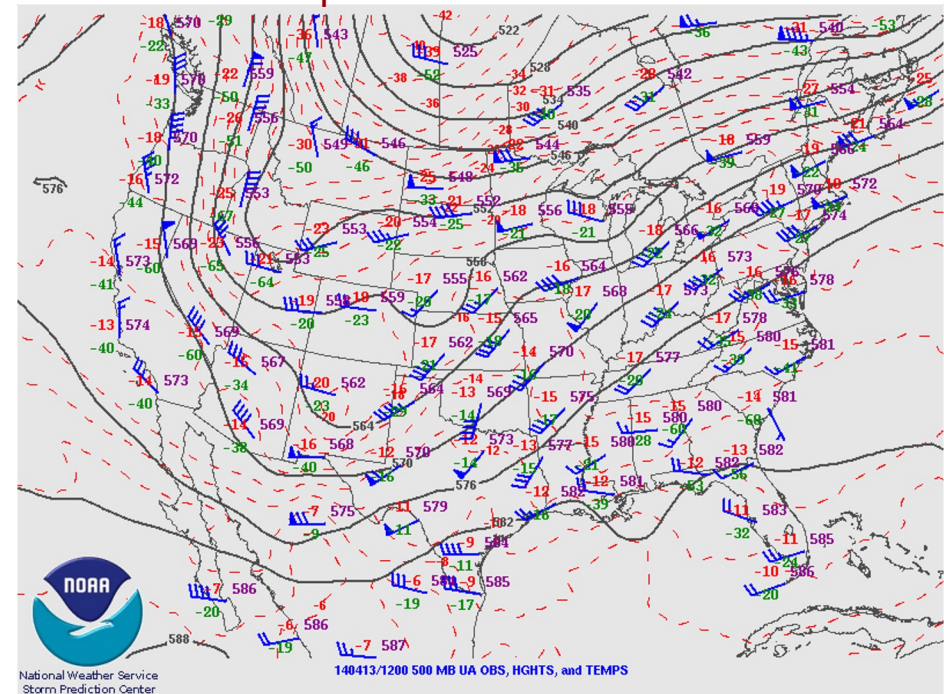
A quick note:

There are alternatives to QG theory (for example, IPV theory), that will work just as well.

We will focus on QG theory in this class for two reasons:

- 1) It's easy to interpret from basic weather charts
- 2) Most U.S. weather entities use QG theory (not the case elsewhere...)

What will this pattern look like in 12-24 hours?



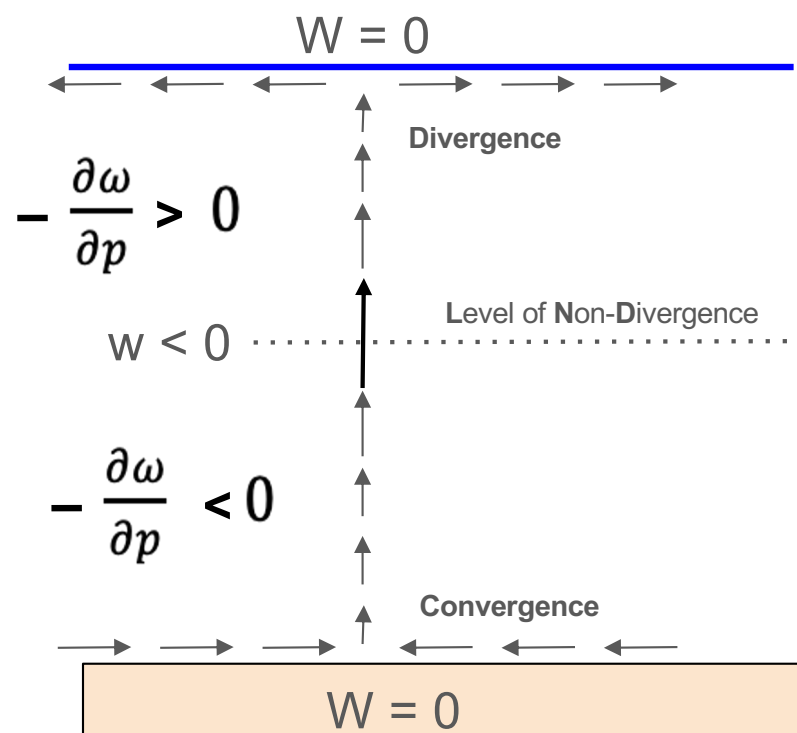
Some Background Concepts

Mass Continuity:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial \omega}{\partial p} = 0$$

We're going to assume:

- On synoptic scales, the troposphere is incompressible.
- Hydrostatic approximation applies
- Vertical velocity is zero at the surface and at the tropopause.
- **Because of mass continuity, any vertical motion is associated with horizontal convergence and divergence**



Some Background Concepts

1.4 thermal wind balance

$$(1) u_g = -\frac{g}{f} \frac{\partial Z}{\partial y} \quad \text{geostrophic wind}$$

$$(2) \frac{\partial Z}{\partial p} = -\frac{RT}{gp} \quad \text{hypsometric eqn}$$

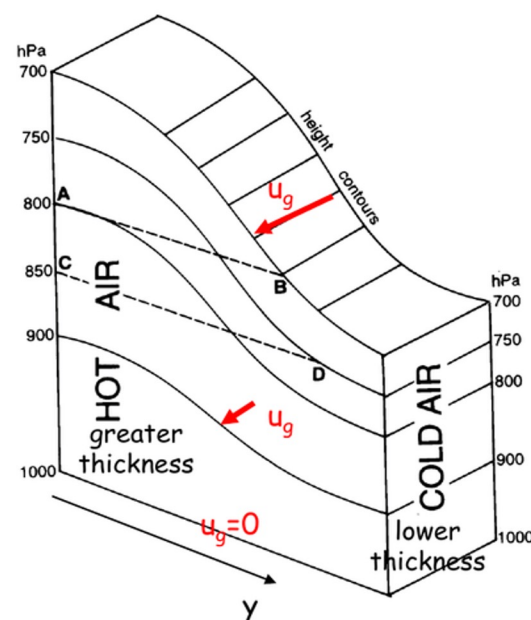
plug (2) into (1)

$$\begin{aligned} \frac{\partial u_g}{\partial p} &= \frac{g}{f} \frac{\partial \left(\frac{RT}{gp} \right)}{\partial y} \\ &= \frac{R}{fp} \frac{\partial T}{\partial y} \end{aligned}$$

finite difference expression:

$$\Delta u_g = \frac{R}{f} \frac{\Delta p}{p} \frac{\Delta \bar{T}}{\Delta y} \quad \text{this is the thermal wind: an increase in wind with height due to a temperature gradient}$$

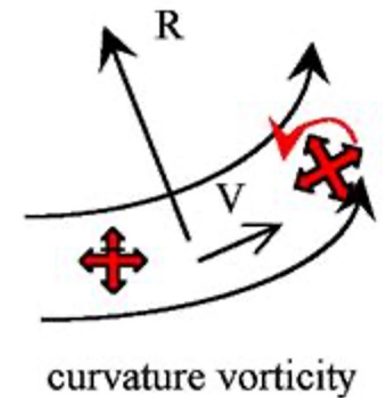
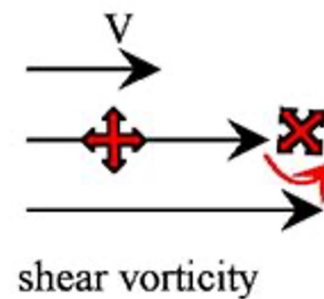
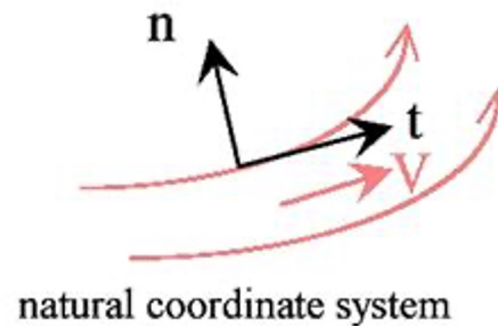
The thermal wind blows ccw around cold pools in the same way as the geostrophic wind blows ccw around lows. The thermal wind is proportional to the T gradient, while the geostrophic wind is proportional to the pressure (or height) gradient.



Some Background Concepts

Vorticity:

- Vorticity is the curl of the wind field
- Exists in all 3 dimensions, but for today we'll only consider the X/Y dimensions
- Vorticity can be generated by curvature in the flow and/or speed shear in the flow
- Vorticity is directly related to vertical motions and convergence/divergence due to the conservation of angular momentum and conservation of mass



Physical Intuition

Charles's Law

- The volume of a gas is directly proportional to the temperature of the gas at a constant pressure.
- If the gas heats up -> it expands!
- If the gas cools down -> it contracts!



Jacques Charles
(1746-1823).



Physical Intuition

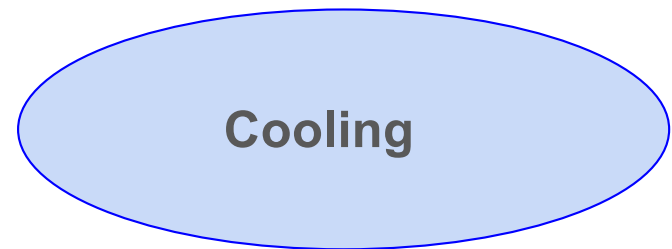


Consider a layer of the atmosphere in contact with the surface:

Warming the layer

Cooling the layer

Initial Height



The ground can't move - so the top of the layer must move!

Physical Intuition



Consider a layer of the atmosphere in contact with the surface:

Warming the layer

Cooling the layer

Final Height

Initial Height

(Expands)

Final Height

(Contracts)

The ground can't move - so the top of the layer must move!

Physical Intuition



Consider a layer of the atmosphere above the surface:

Warming the layer

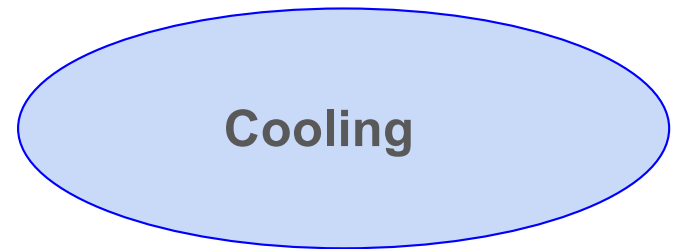
Cooling the layer

Initial Height 1

Warming

Cooling

Initial Height 2



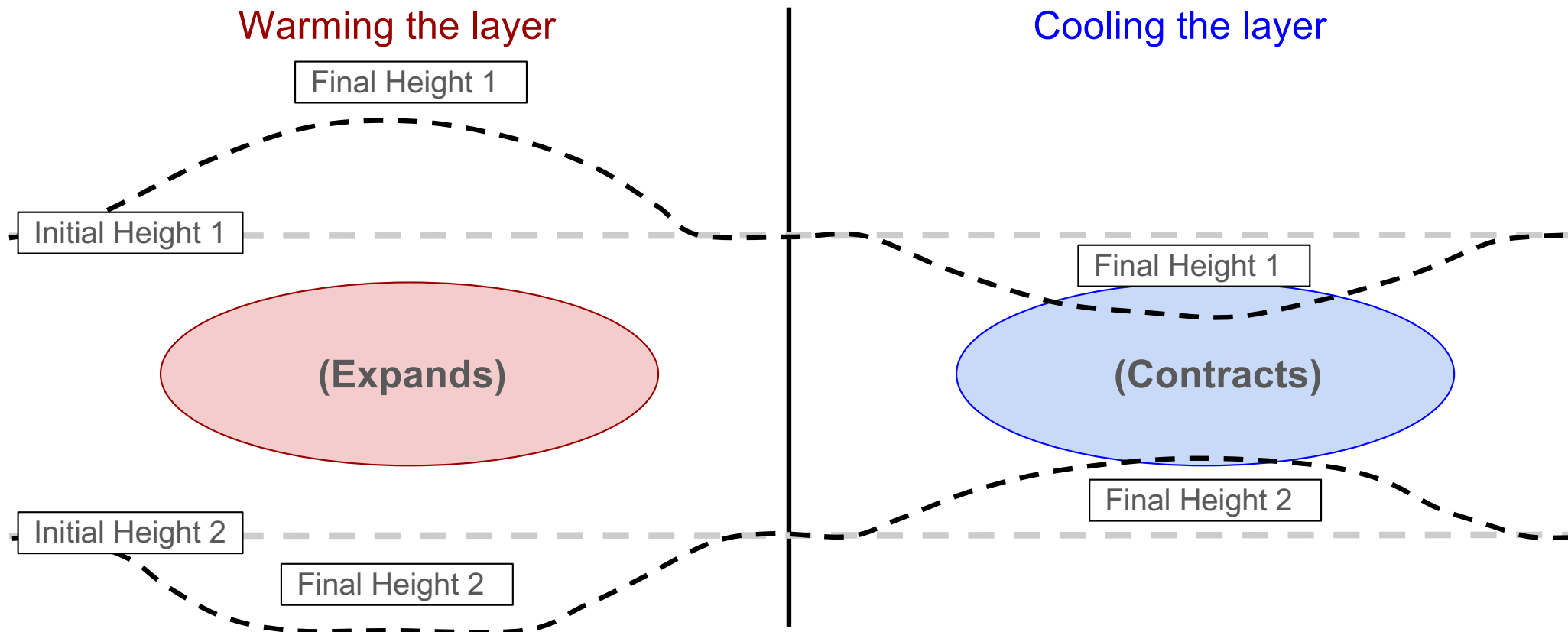
Physical Intuition



Consider a layer of the atmosphere above the surface:

Warming the layer

Cooling the layer



Physical Intuition

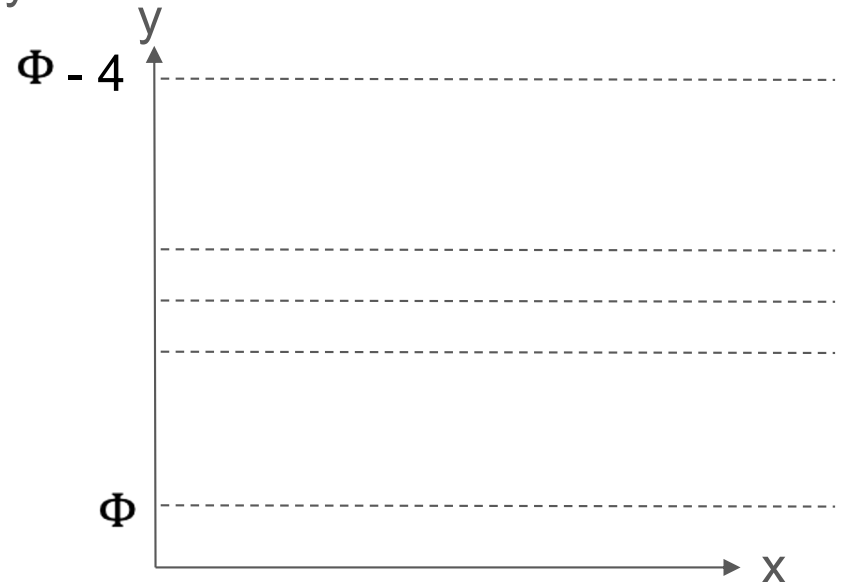
Geostrophic Vorticity:

Plug in geostrophic wind balance into the vorticity equation.

You end up with a form of geostrophic vorticity that relates to the Laplacian of the geopotential height field.

Thus, if you locally change the vorticity at a location, you must also change the geopotential height (i.e. thickness) field!

Consider this geopotential height field.
What would vorticity look like?



Physical Intuition

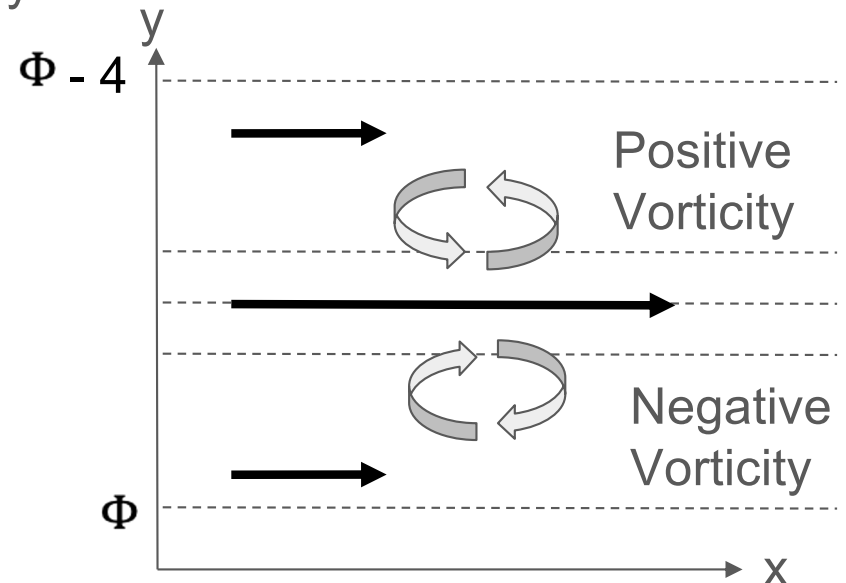
Geostrophic Vorticity:

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Thus, if you locally change the vorticity at a location, you must also change the geopotential height (i.e. thickness) field!

Consider this geopotential height field.
What would vorticity look like?



Use the thermal wind relation to confirm!

Physical Intuition

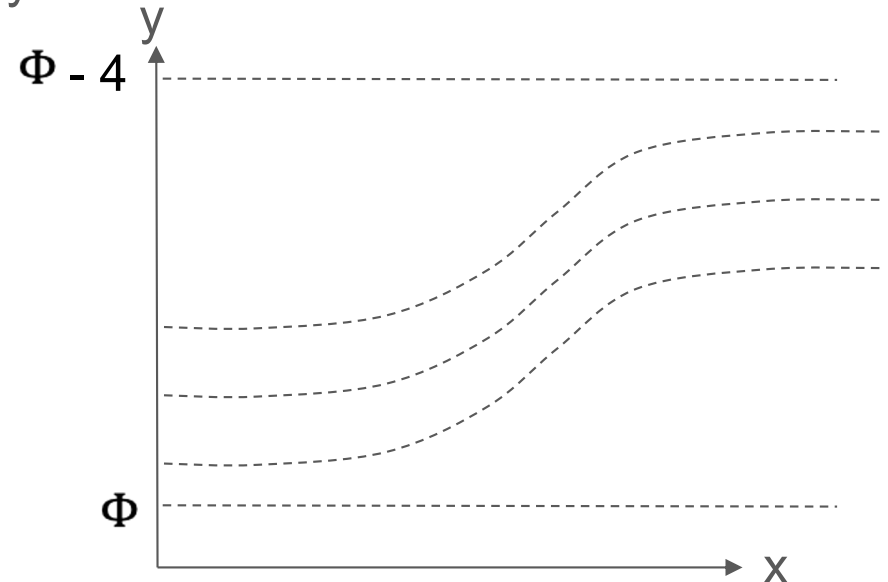
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You end up with a form of geostrophic vorticity that relates to the Laplacian of the geopotential height field.

Thus, if you locally change the vorticity at a location, you must also change the geopotential height (i.e. thickness) field!

Consider this geopotential height field.
What would vorticity look like?



If we change the geopotential height field, how will the vorticity field change (and vice versa)?

QG Vorticity and Thermo Equations

QG vorticity equation

$$\frac{d(\zeta_g + f)}{dt} = f_0 \frac{\partial \omega}{\partial p}$$

Rate of change of absolute vorticity

Vertical motion (or convergence/divergence)

QG thermodynamic equation

$$\frac{dT}{dt} = \frac{p}{R_d} \sigma \omega$$

Rate of change of temperature

Vertical motion (or expansion/contraction from vertical motion)

See Bluestein Vol. 1
page 329 for details



QG Height Tendency Equation

$$\left(\nabla_p^2 + \frac{f_0^2}{\sigma} \frac{\partial^2}{\partial p^2} \right) \chi = -f_0 \mathbf{V}_g \cdot \nabla_p \left(\frac{1}{f_0} \nabla_p^2 \Phi + f \right) - \frac{f_0^2}{\sigma} \frac{\partial}{\partial p} \left[\mathbf{V}_g \cdot \nabla_p \left(-\frac{\partial \Phi}{\partial p} \right) \right] - \frac{\partial H}{\partial p}$$

2nd derivative operator



Absolute vorticity advection



Differential thermal
advection



Diabatic heating



QG Height Tendency Equation

$$\underbrace{\left(\nabla_p^2 + \frac{f_0^2}{\sigma} \frac{\partial^2}{\partial p^2}\right)}_A \chi = \underbrace{-f_0 \mathbf{V}_g \cdot \nabla_p \left(\frac{1}{f_0} \nabla_p^2 \Phi + f\right)}_B - \underbrace{\frac{f_0^2}{\sigma} \frac{\partial}{\partial p} \left[\mathbf{V}_g \cdot \nabla_p \left(-\frac{\partial \Phi}{\partial p}\right)\right]}_C - \underbrace{\frac{\partial H}{\partial p}}_D \quad \chi \equiv \frac{\partial \Phi}{\partial t}$$

- Height change (A) = B + C + D
- Term B: advection of geostrophic absolute vorticity by the geostrophic wind
 - Cyclonic vorticity advection (CVA) \equiv height falls
 - Propagation mechanism for troughs and ridges
- Term C: differential advection of thickness by the geostrophic wind
 - Referred to as thermal advection or temperature advection
 - Heights rise above and fall below level of maximum warm advection
 - Heights fall above and rise below level of maximum cold advection
 - Amplification mechanism for troughs and ridges
- Term D: differential diabatic heating
 - Heights rise above and fall below level of maximum latent heating
 - Heights fall above and rise below level of maximum radiational cooling

QG X Breakdown

2nd Derivative Operator:

$$\left(\nabla_p^2 + \frac{f_0^2}{\sigma} \frac{\partial^2}{\partial p^2} \right) \chi$$

On synoptic scales, we can roughly assume that the height field (and thus the height tendency field) is sinusoidal.

Take the second derivative of a sine function:

$$d(d(\sin(x)))$$

$$= d(\cos(x))$$

$$= -\sin(x)$$

**This assumption turns the
2nd derivative operation
into a simple minus sign!**



QG X Breakdown

$$-f_0 \mathbf{V}_g \cdot \nabla_p \left(\frac{1}{f_0} \nabla_p^2 \Phi + f \right)$$

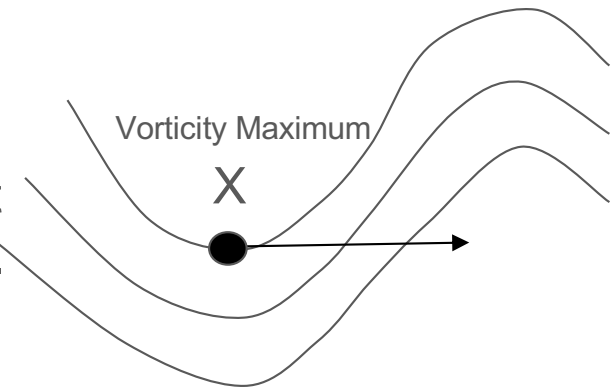
OR

$$-f_0 \mathbf{V}_g \cdot \nabla_p (\zeta_g + f)$$

Consider this case:

Advection of absolute vorticity:

- This considers both relative vorticity (related to the height field) and planetary vorticity (related to the Coriolis force).
- Here we see how moving the height field (more specifically, the Laplacian of the height field) can change the height field.



$$\begin{aligned} V_g &= 0 \\ U_g &> 0 \end{aligned}$$

QG X Breakdown

$$-f_0 \mathbf{V}_g \cdot \nabla_p \left(\frac{1}{f_0} \nabla_p^2 \Phi + f \right)$$

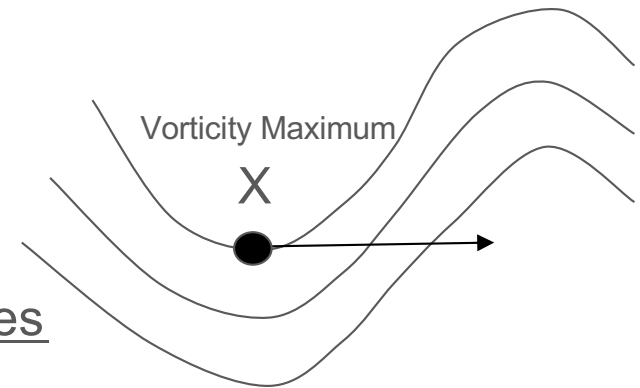
OR

$$-f_0 \mathbf{V}_g \cdot \nabla_p (\zeta_g + f)$$

Consider this case:

Advection of absolute vorticity:

- Advecting cyclonic vorticity (CVA) leads to height falls
- Advecting anticyclonic vorticity (AVA) leads to height rises



$$\begin{aligned} V_g &= 0 \\ U_g &> 0 \end{aligned}$$

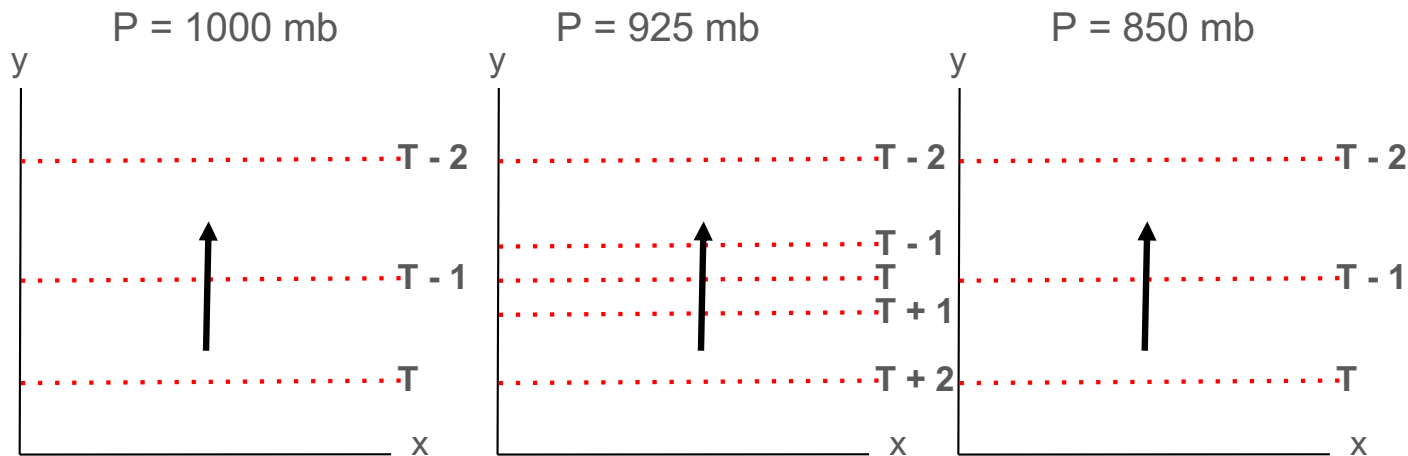
QG X Breakdown

$$-\frac{f_0^2}{\sigma} \frac{\partial}{\partial p} \left[\mathbf{V}_g \cdot \nabla_p \left(-\frac{\partial \Phi}{\partial p} \right) \right]$$

OR

$$-\frac{f_0^2}{\sigma} \frac{\partial}{\partial p} \left[-\frac{R}{p} \mathbf{V}_g \cdot \nabla_p T \right]$$

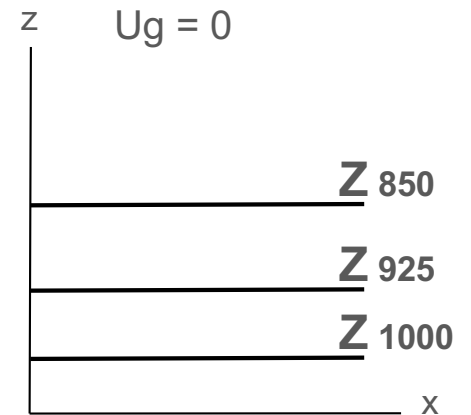
Differential Thermal Advection



Assume:

$$V_g > 0$$

$$U_g = 0$$



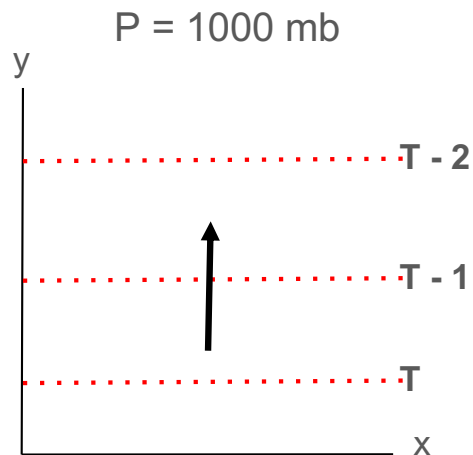
QG X Breakdown

$$-\frac{f_0^2}{\sigma} \frac{\partial}{\partial p} \left[\mathbf{V}_g \cdot \nabla_p \left(-\frac{\partial \Phi}{\partial p} \right) \right]$$

OR

$$-\frac{f_0^2}{\sigma} \frac{\partial}{\partial p} \left[-\frac{R}{p} \mathbf{V}_g \cdot \nabla_p T \right]$$

Differential Thermal Advection



Consider only y component of thermal advection:

$$V_g > 0$$

$$dT/dy < 0$$

$$-V_g(dT/dy) > 0$$

Assume:

$$V_g > 0$$

$$U_g = 0$$

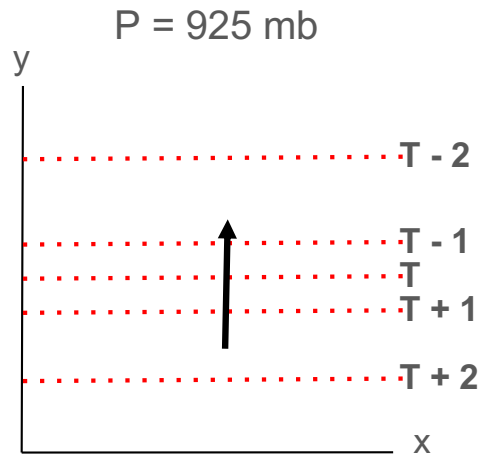
QG X Breakdown

$$-\frac{f_0^2}{\sigma} \frac{\partial}{\partial p} \left[\mathbf{V}_g \cdot \nabla_p \left(-\frac{\partial \Phi}{\partial p} \right) \right]$$

OR

$$-\frac{f_0^2}{\sigma} \frac{\partial}{\partial p} \left[-\frac{R}{p} \mathbf{V}_g \cdot \nabla_p T \right]$$

Differential Thermal Advection



Consider only y component of thermal advection:

$$V_g > 0$$

$$dT/dy \ll 0$$

$$-V_g(dT/dy) \gg 0$$

Assume:

$$V_g > 0$$

$$U_g = 0$$

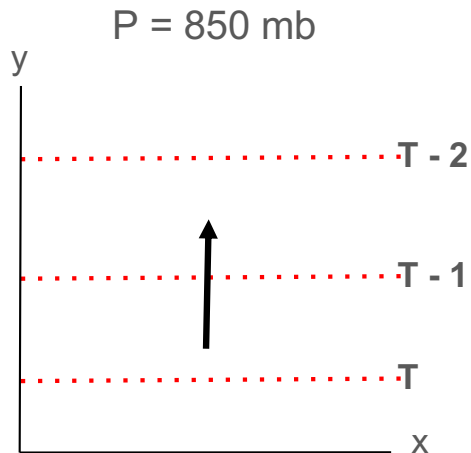
QG X Breakdown

$$-\frac{f_0^2}{\sigma} \frac{\partial}{\partial p} \left[\mathbf{V}_g \cdot \nabla_p \left(-\frac{\partial \Phi}{\partial p} \right) \right]$$

OR

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Differential Thermal Advection



Consider only y component of thermal advection:

$$V_g > 0$$

$$dT/dy < 0$$

$$-V_g(dT/dy) > 0$$

Assume:

$$V_g > 0$$

$$U_g = 0$$

QG X Breakdown

$$\boxed{-\frac{f_0^2}{\sigma} \frac{\partial}{\partial p} \left[\mathbf{V}_g \cdot \nabla_p \left(-\frac{\partial \Phi}{\partial p} \right) \right]}$$

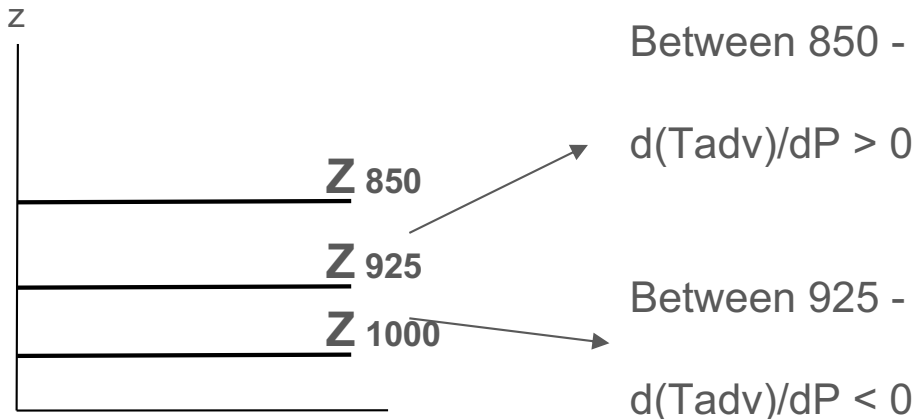
OR

$$-\frac{f_0^2}{\sigma} \frac{\partial}{\partial p} \left[-\frac{R}{p} \mathbf{V}_g \cdot \nabla_p T \right]$$

Can't forget this -1!

Differential Thermal Advection

Now consider differential portion between the pressure levels:



Between 850 - 925:

$$d(T_{adv})/dP > 0$$

Between 925 - 1000:

$$d(T_{adv})/dP < 0$$

$$-\chi \propto -1 * (\text{positive value}) = \text{positive value}$$

$$-\chi \propto -1 * (\text{negative value}) = \text{negative value}$$

QG X Breakdown

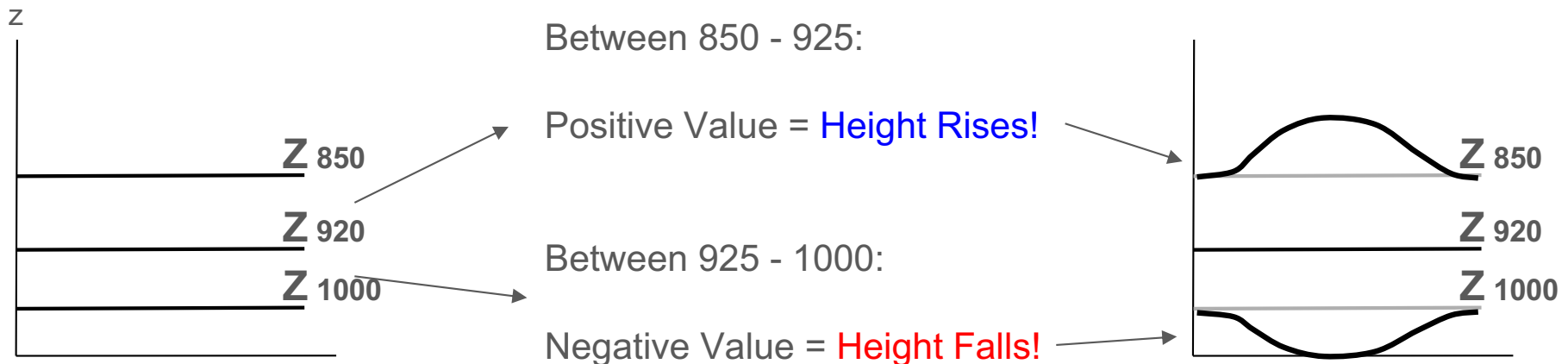
$$-\frac{f_0^2}{\sigma} \frac{\partial}{\partial p} \left[\mathbf{V}_g \cdot \nabla_p \left(-\frac{\partial \Phi}{\partial p} \right) \right]$$

OR

$$-\frac{f_0^2}{\sigma} \frac{\partial}{\partial p} \left[-\frac{R}{p} \mathbf{V}_g \cdot \nabla_p T \right]$$

Differential Thermal Advection

Now consider differential portion between the pressure levels:



QG X Breakdown

$$-\frac{f_0^2}{\sigma} \frac{\partial}{\partial p} \left[\mathbf{V}_g \cdot \nabla_p \left(-\frac{\partial \Phi}{\partial p} \right) \right]$$

OR

$$-\frac{f_0^2}{\sigma} \frac{\partial}{\partial p} \left[-\frac{R}{p} \mathbf{V}_g \cdot \nabla_p T \right]$$

Differential Thermal Advection:

- **Warm air advection** at a given pressure level induces height rises above that level, and height falls below that level
- **Cold air advection** does the opposite: it causes height falls above the given pressure level, and height rises below.
- This ties back directly to Charles's Law!



QG X Breakdown

$$-\frac{\partial H}{\partial p}$$

Diabatic Heating:

- Similar in concept to differential thermal advection:
Diabatic heating in a layer causes the layer to expand;
Diabatic cooling in layer causes the layer to contract.
- Causes height rises (falls) above (below) the source of heating.
- Causes height rises (falls) below (above) the source of cooling
- Examples:
 - Latent heat release from large systems (hurricanes, large cyclones, etc...)
 - Mesohighs in the wake of strong MCSs

QG X Breakdown



$$-\frac{\partial H}{\partial p}$$

Warming the layer

Cooling the layer

Final Height 1

Initial Height 1

Heating source

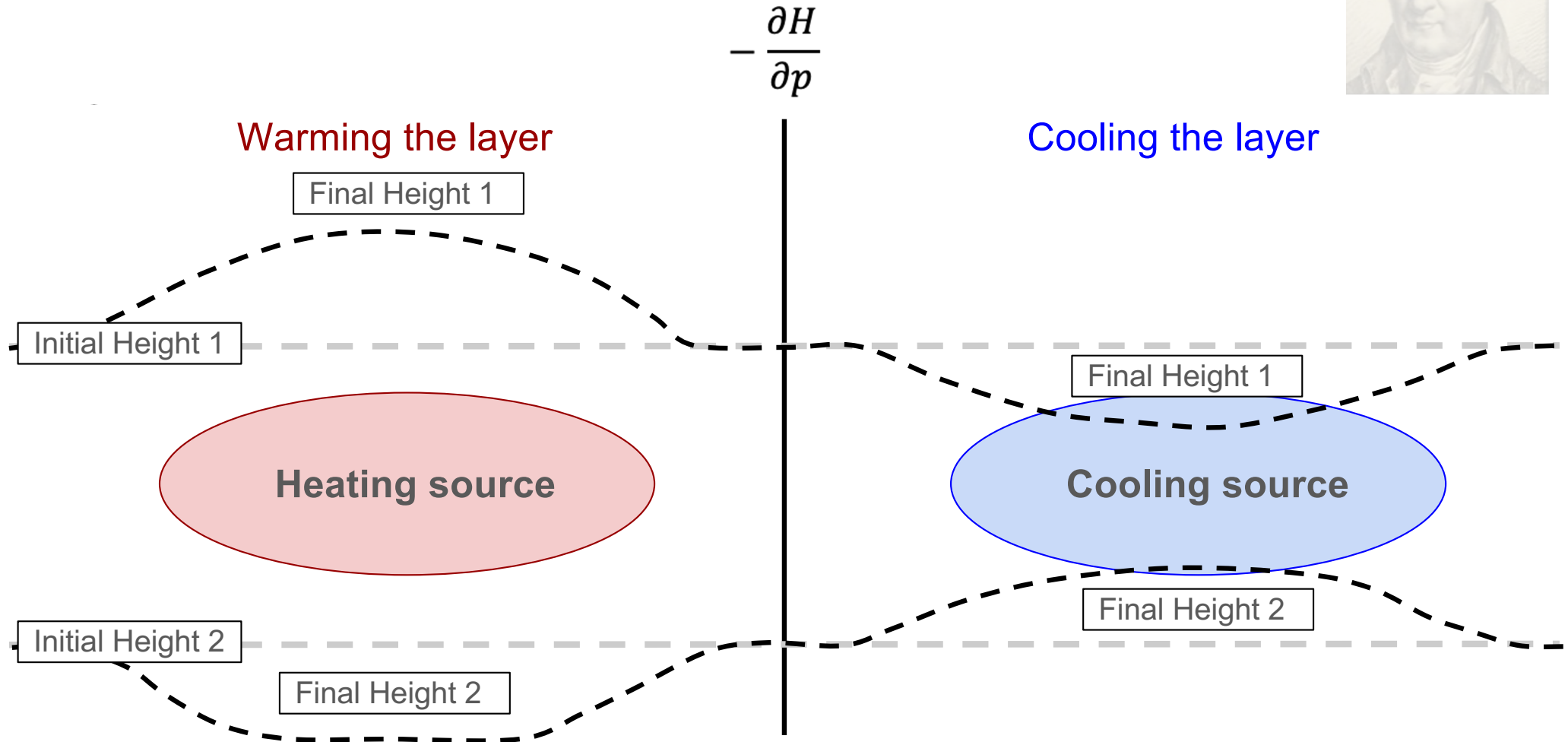
Final Height 1

Cooling source

Initial Height 2

Final Height 2

Final Height 2



QG X Application

$$\left(\nabla_p^2 + \frac{f_0^2}{\sigma} \frac{\partial^2}{\partial p^2}\right) \chi = -f_0 \mathbf{V}_g \cdot \nabla_p \left(\frac{1}{f_0} \nabla_p^2 \Phi + f\right) - \frac{f_0^2}{\sigma} \frac{\partial}{\partial p} \left[\mathbf{V}_g \cdot \nabla_p \left(-\frac{\partial \Phi}{\partial p}\right)\right] - \frac{\partial H}{\partial p}$$

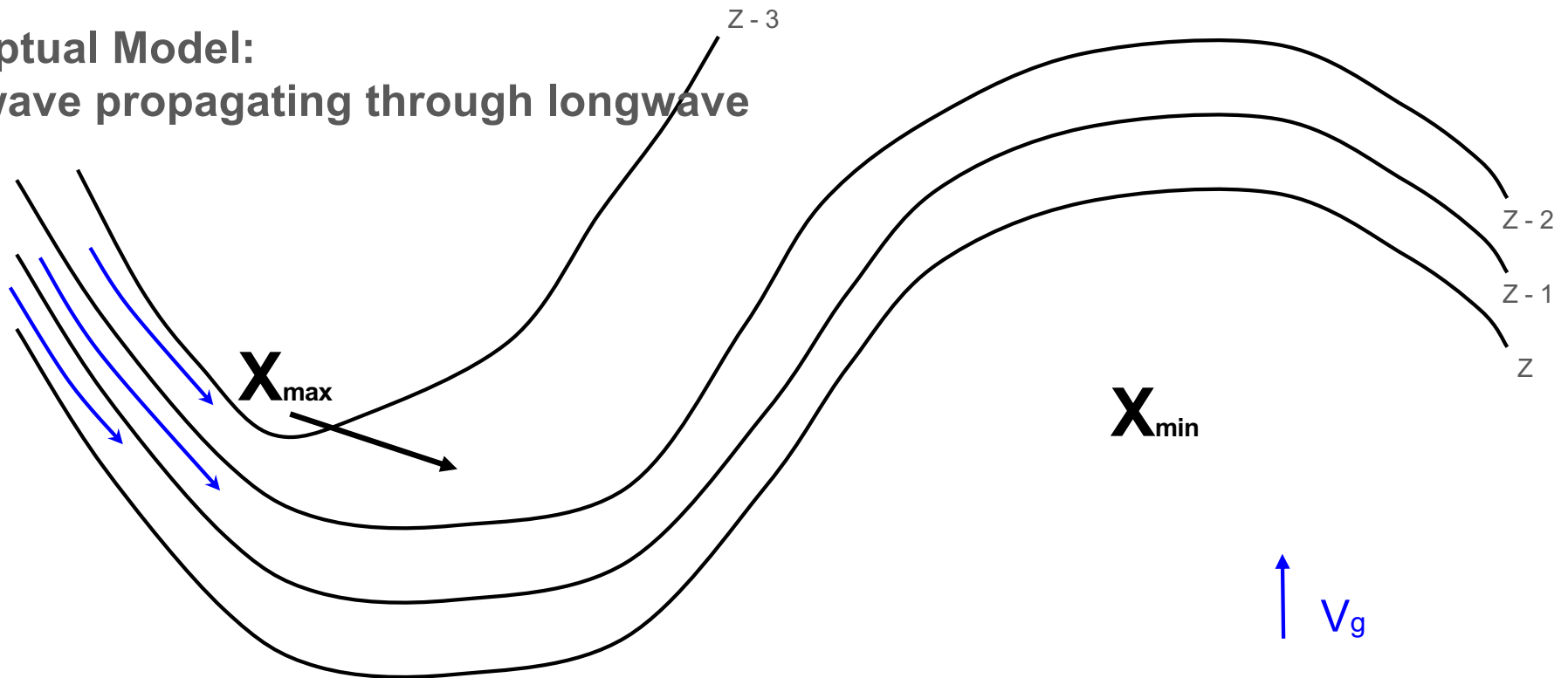
How do we use this?

- Use the vorticity advection term to help anticipate where a vorticity maximum or minimum (i.e. a trough or a ridge) will go.
- Use the differential thermal advection term to anticipate whether or not a trough or ridge will amplify.
- The diabatic heating term is similar to the thermal advection term, but is typically not as consequential.

QG X Application

$$\left(\nabla_p^2 + \frac{f_0^2}{\sigma} \frac{\partial^2}{\partial p^2} \right) \chi = -f_0 \mathbf{V}_g \cdot \nabla_p \left(\frac{1}{f_0} \nabla_p^2 \Phi + f \right) - \frac{f_0^2}{\sigma} \frac{\partial}{\partial p} \left[\mathbf{V}_g \cdot \nabla_p \left(-\frac{\partial \Phi}{\partial p} \right) \right] - \frac{\partial H}{\partial p}$$

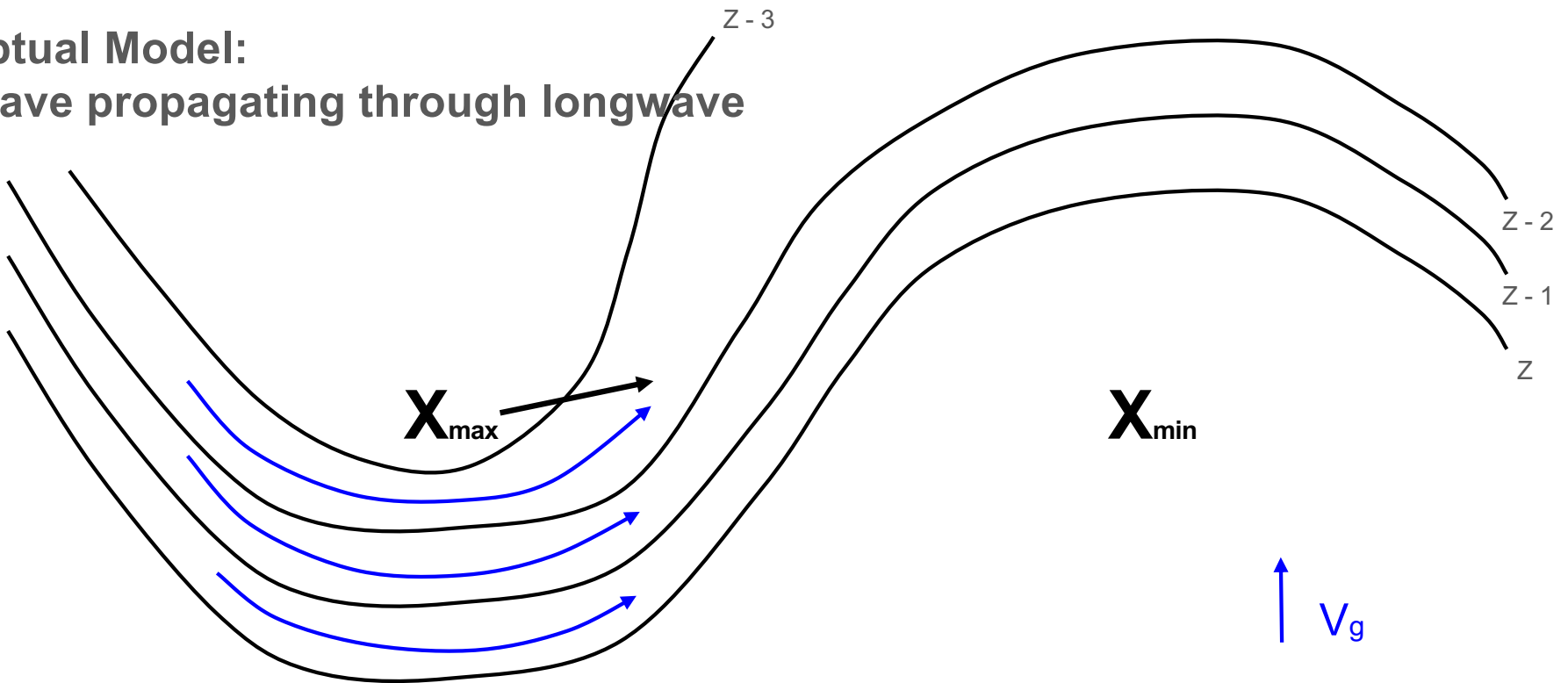
Conceptual Model:
Shortwave propagating through longwave



QG X Application

$$\left(\nabla_p^2 + \frac{f_0^2}{\sigma} \frac{\partial^2}{\partial p^2} \right) \chi = -f_0 \mathbf{V}_g \cdot \nabla_p \left(\frac{1}{f_0} \nabla_p^2 \Phi + f \right) - \frac{f_0^2}{\sigma} \frac{\partial}{\partial p} \left[\mathbf{V}_g \cdot \nabla_p \left(-\frac{\partial \Phi}{\partial p} \right) \right] - \frac{\partial H}{\partial p}$$

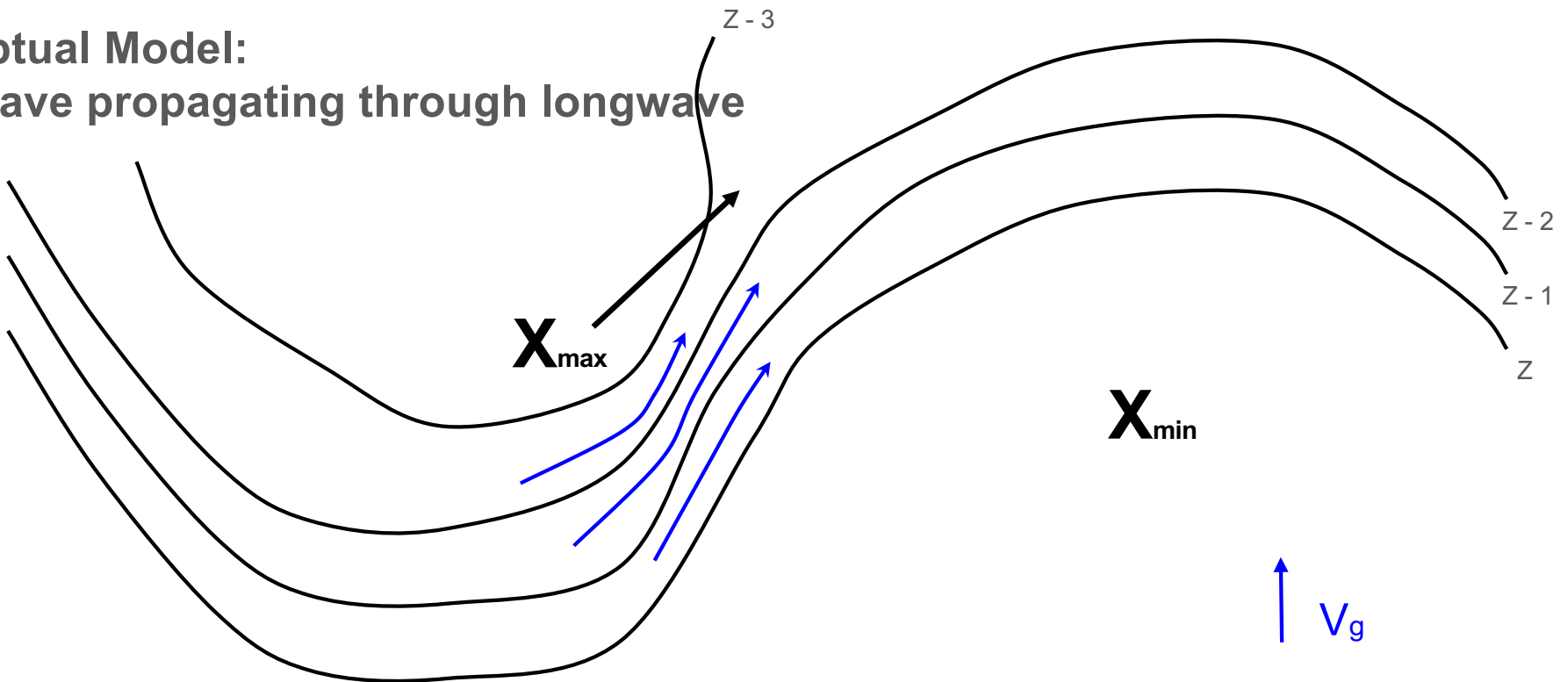
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QG X Application

$$\left(\nabla_p^2 + \frac{f_0^2}{\sigma} \frac{\partial^2}{\partial p^2} \right) \chi = -f_0 \mathbf{V}_g \cdot \nabla_p \left(\frac{1}{f_0} \nabla_p^2 \Phi + f \right) - \frac{f_0^2}{\sigma} \frac{\partial}{\partial p} \left[\mathbf{V}_g \cdot \nabla_p \left(-\frac{\partial \Phi}{\partial p} \right) \right] - \frac{\partial H}{\partial p}$$

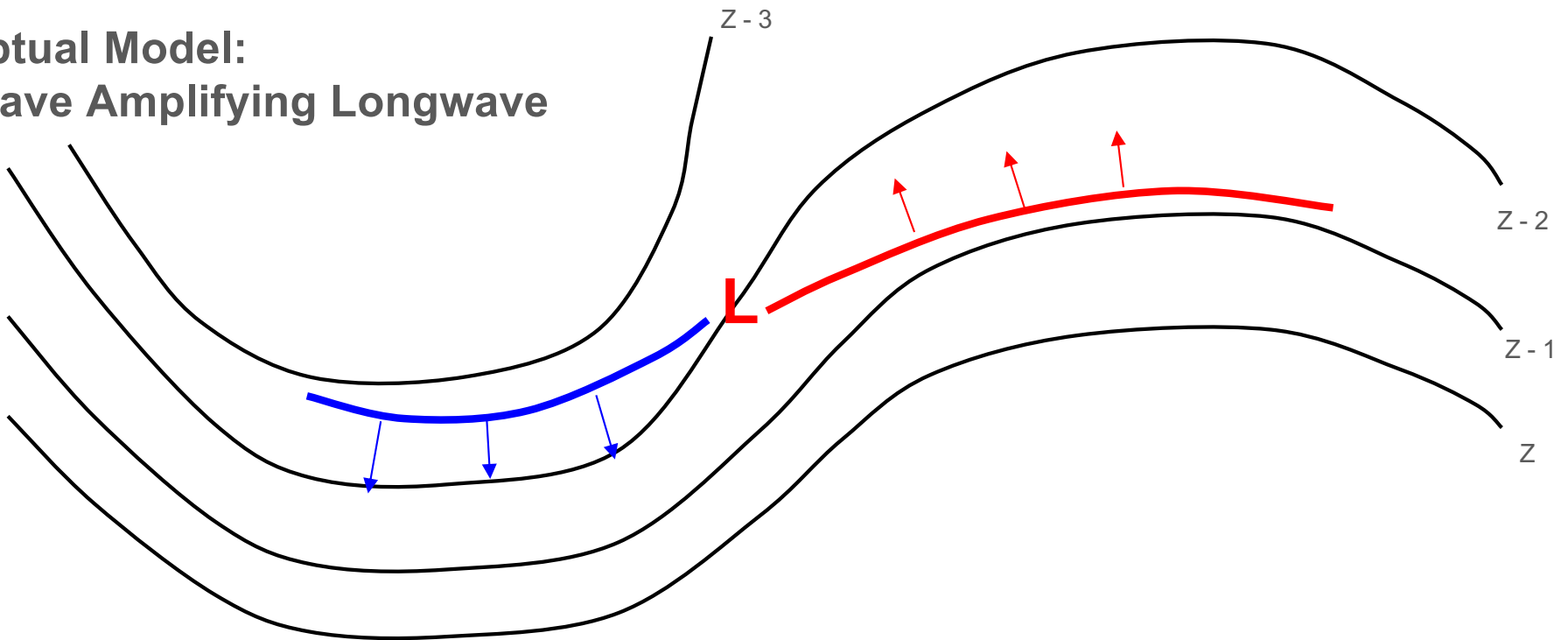
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QG X Application

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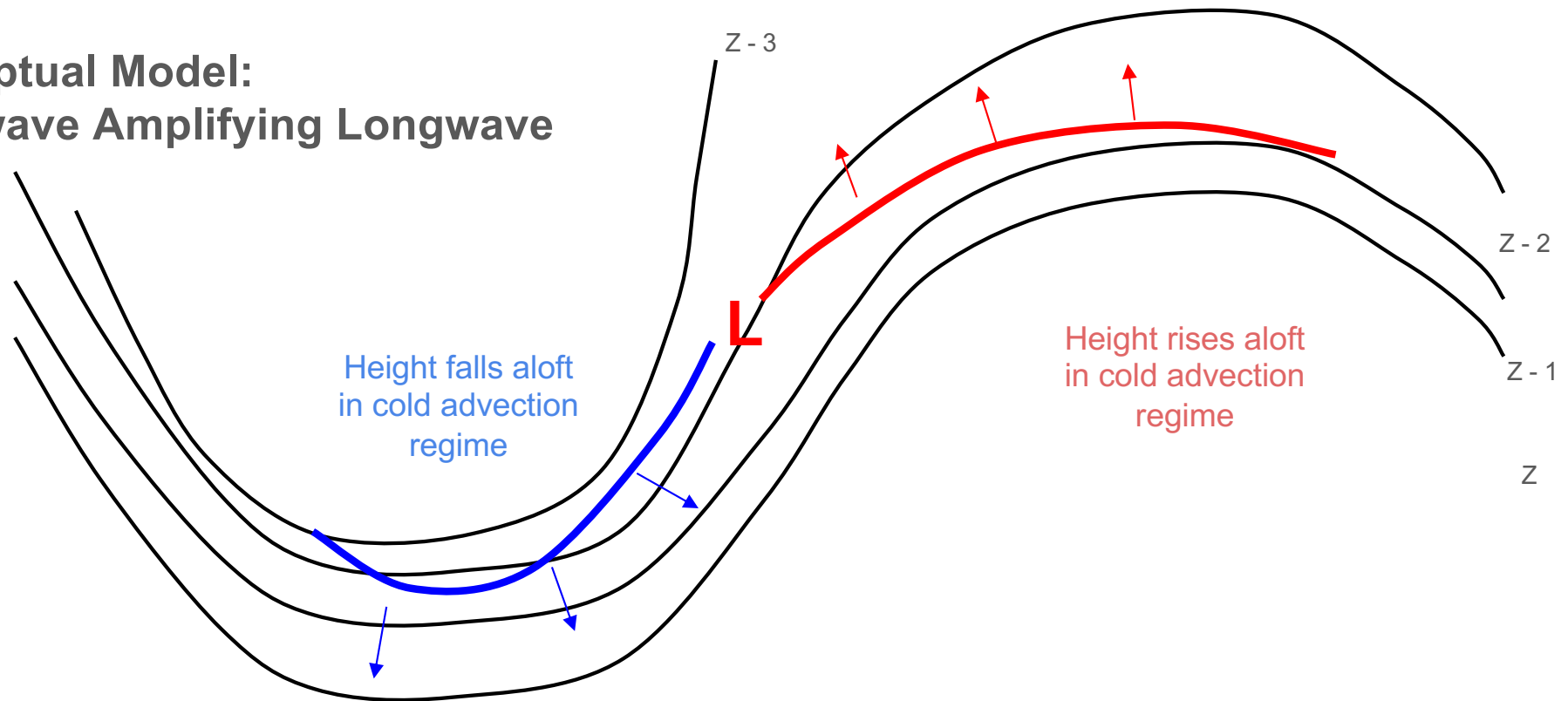
**Conceptual Model:
Shortwave Amplifying Longwave**



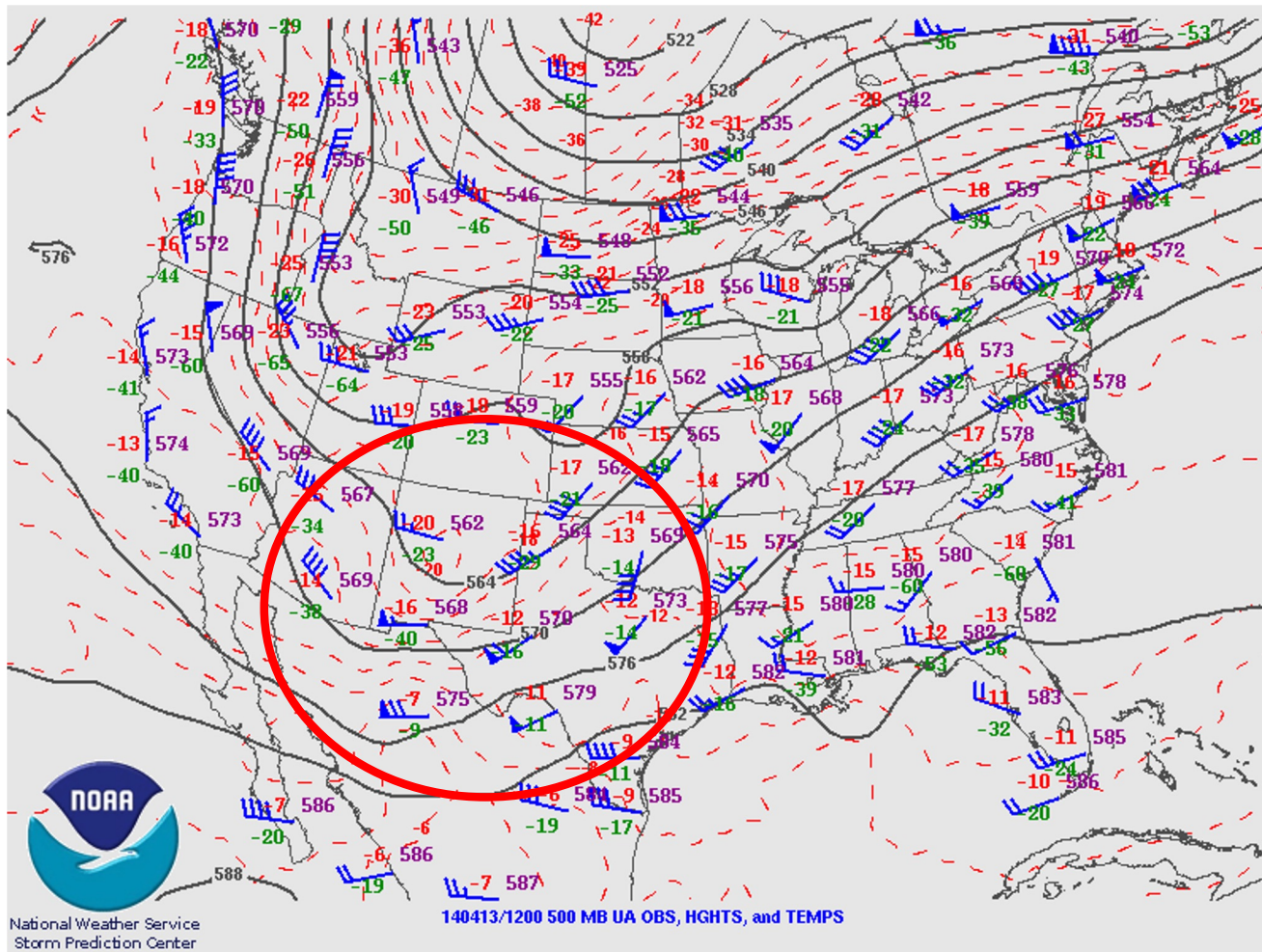
QG X Application

$$\left(\nabla_p^2 + \frac{f_0^2}{\sigma} \frac{\partial^2}{\partial p^2} \right) \chi = -f_0 \mathbf{V}_g \cdot \nabla_p \left(\frac{1}{f_0} \nabla_p^2 \Phi + f \right) - \frac{f_0^2}{\sigma} \frac{\partial}{\partial p} \left[\mathbf{V}_g \cdot \nabla_p \left(-\frac{\partial \Phi}{\partial p} \right) \right] - \frac{\partial H}{\partial p}$$

**Conceptual Model:
Shortwave Amplifying Longwave**



QG X Examples

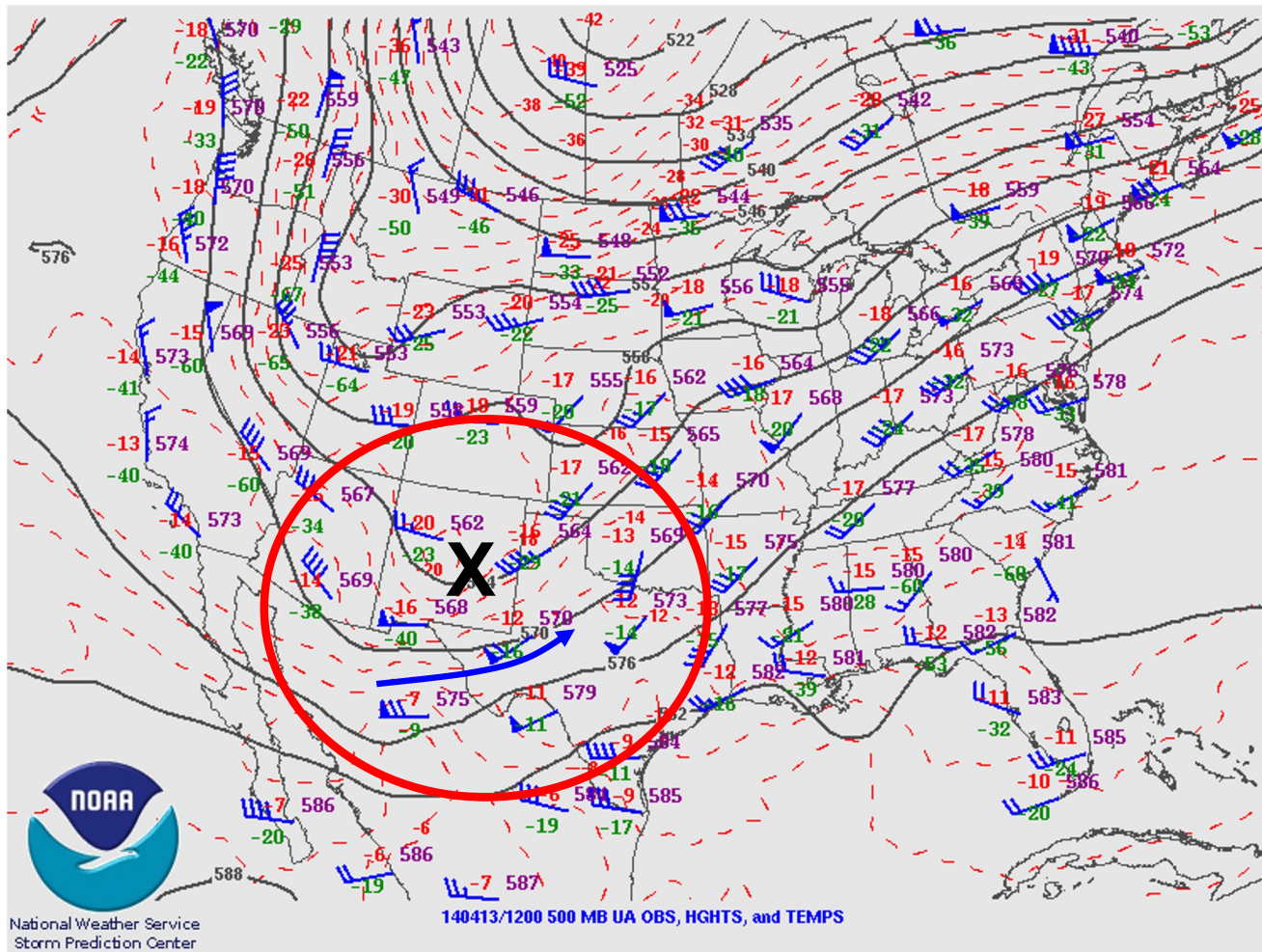


Focus on the shortwave trough in New Mexico.

Where is the vorticity maximum at?

Where will the 500 mb winds advect this vort. max?

QG X Examples

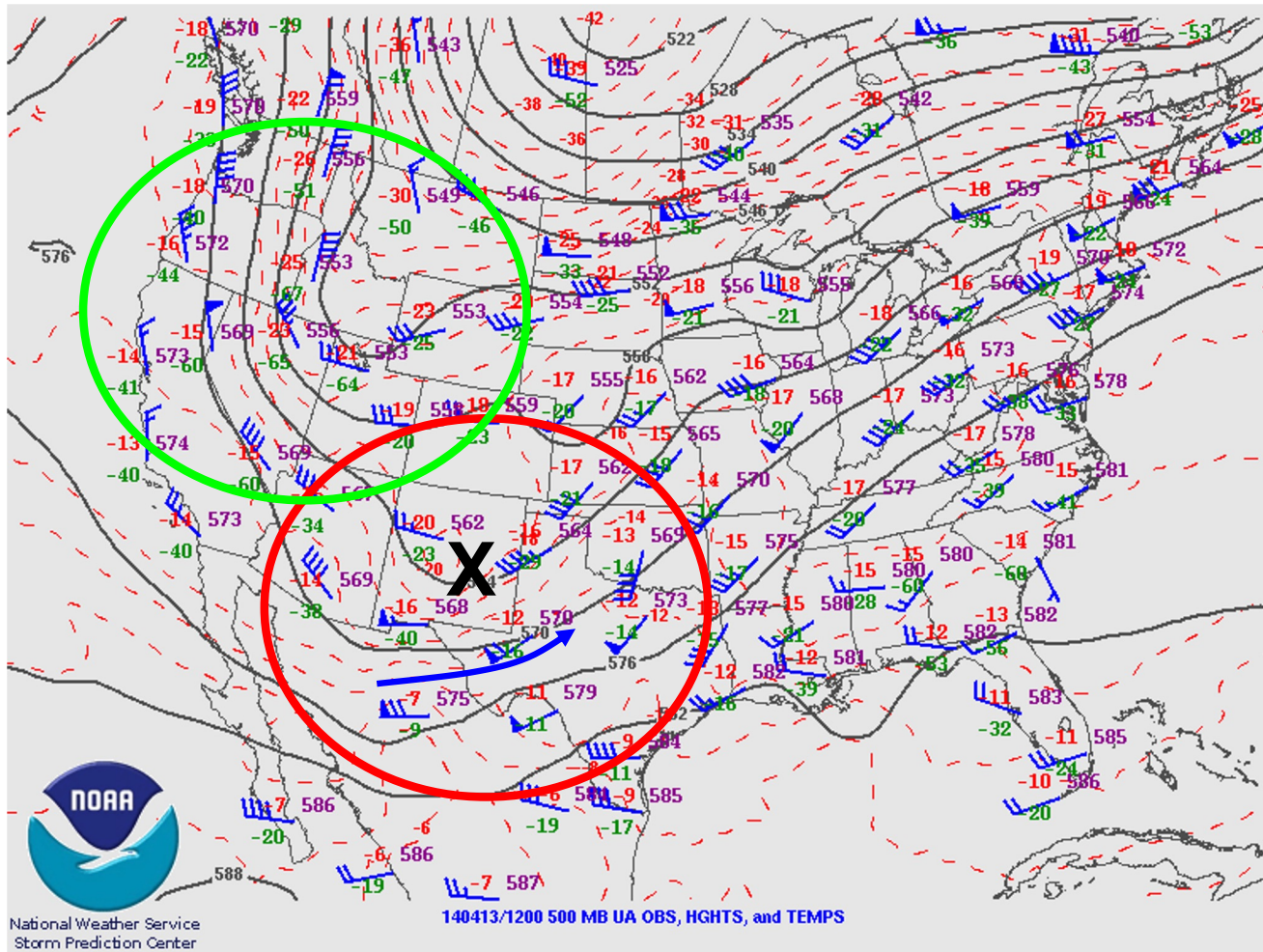


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QG X Examples

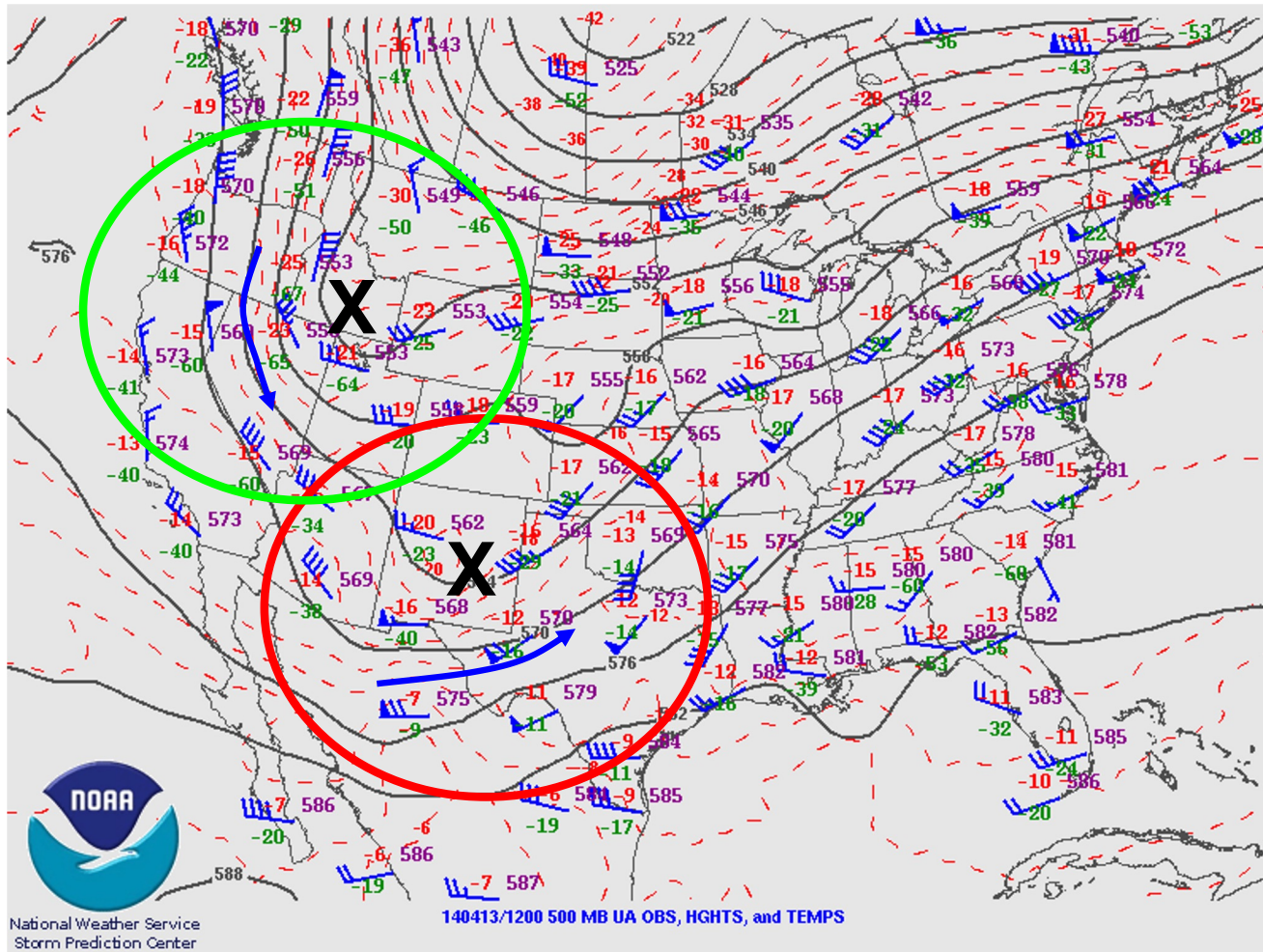


How about this shortwave over the northern Great Basin?

Where is the vorticity maximum?

Where will the geostrophic winds advect the vorticity?

QG X Examples

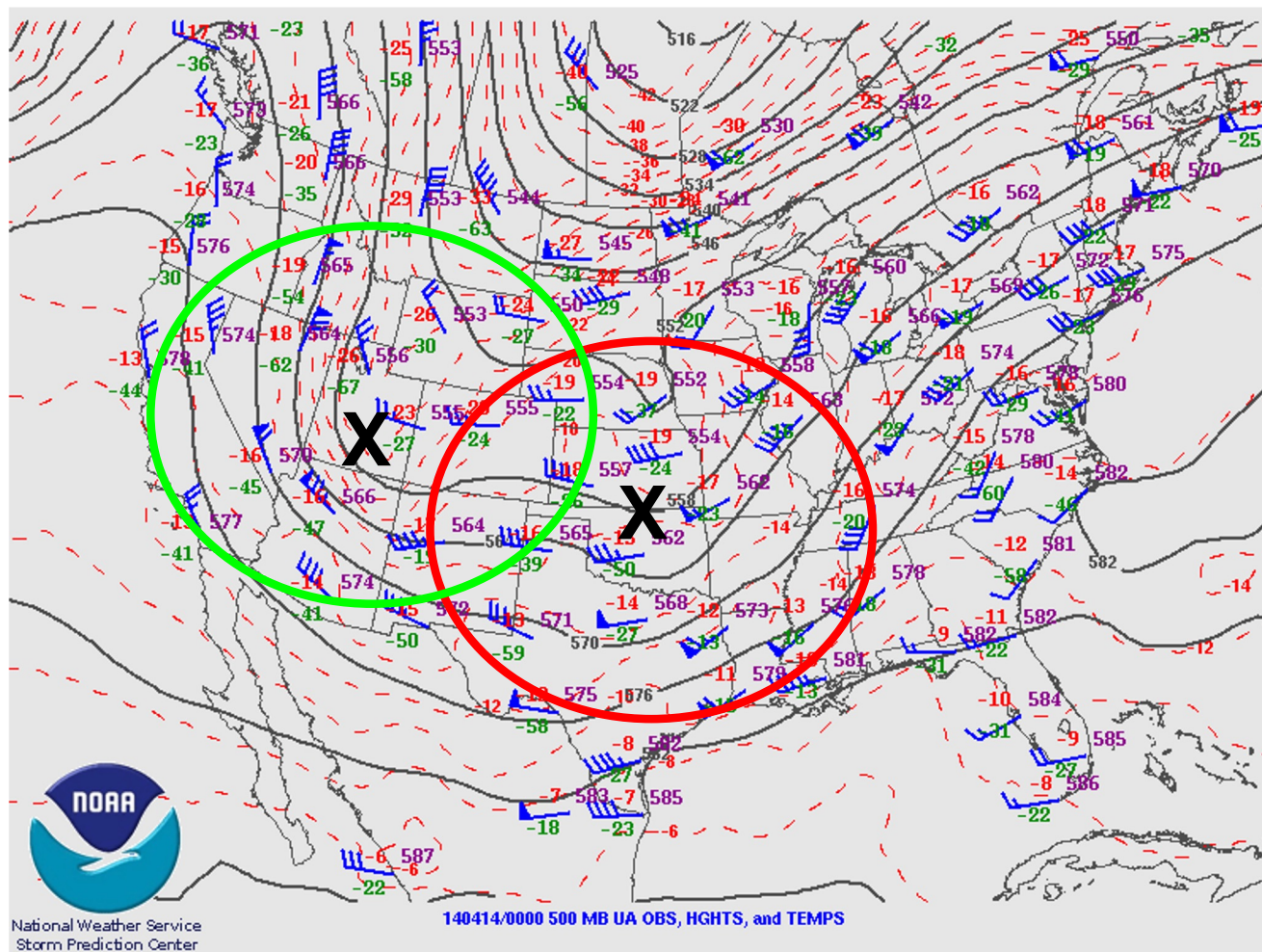


How about this shortwave over the northern Great Basin?

Where is the vorticity maximum?

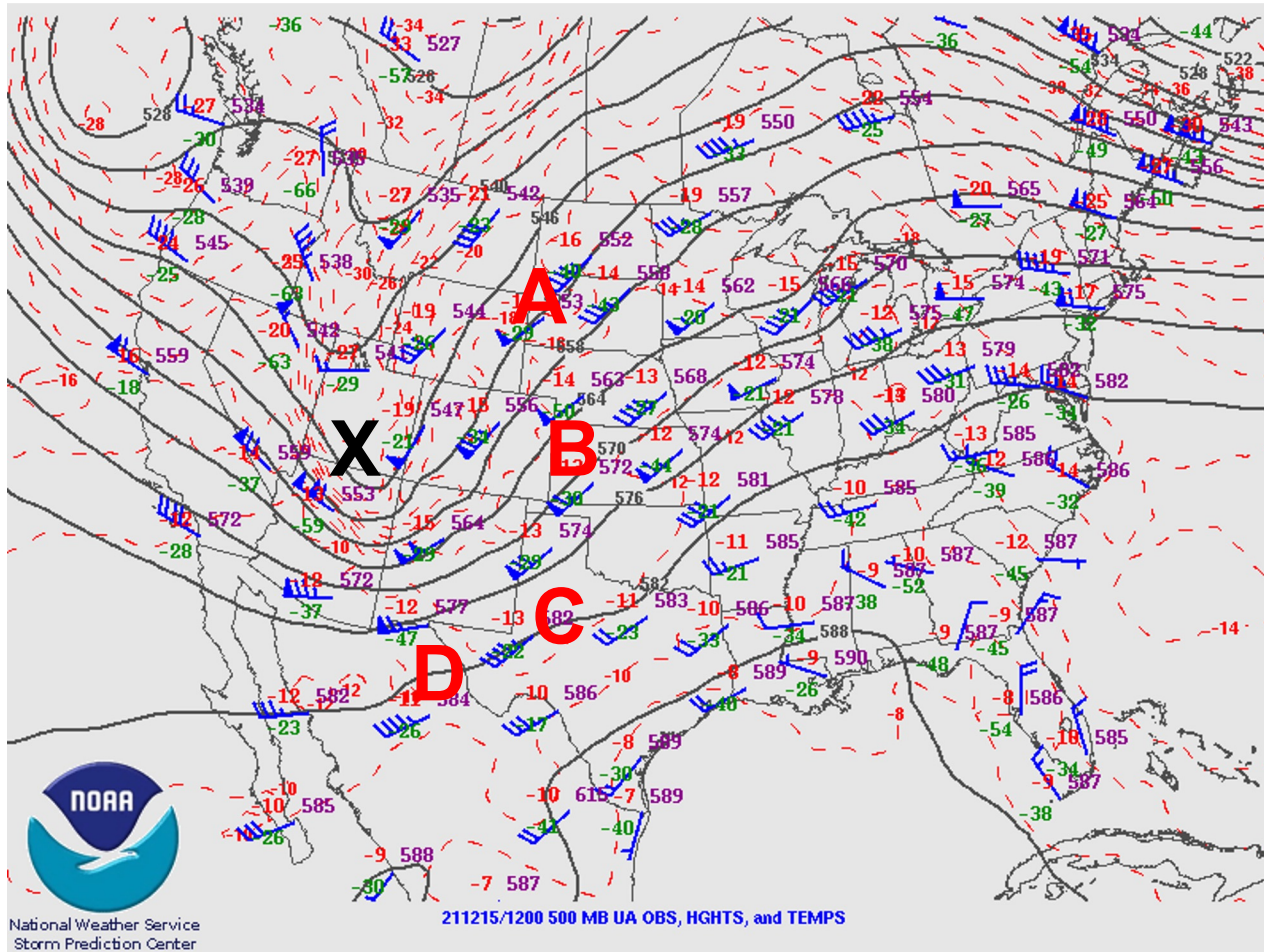
Where will the geostrophic winds advect the vorticity?

QG X Examples



Did this meet your expectations?

QG X Examples

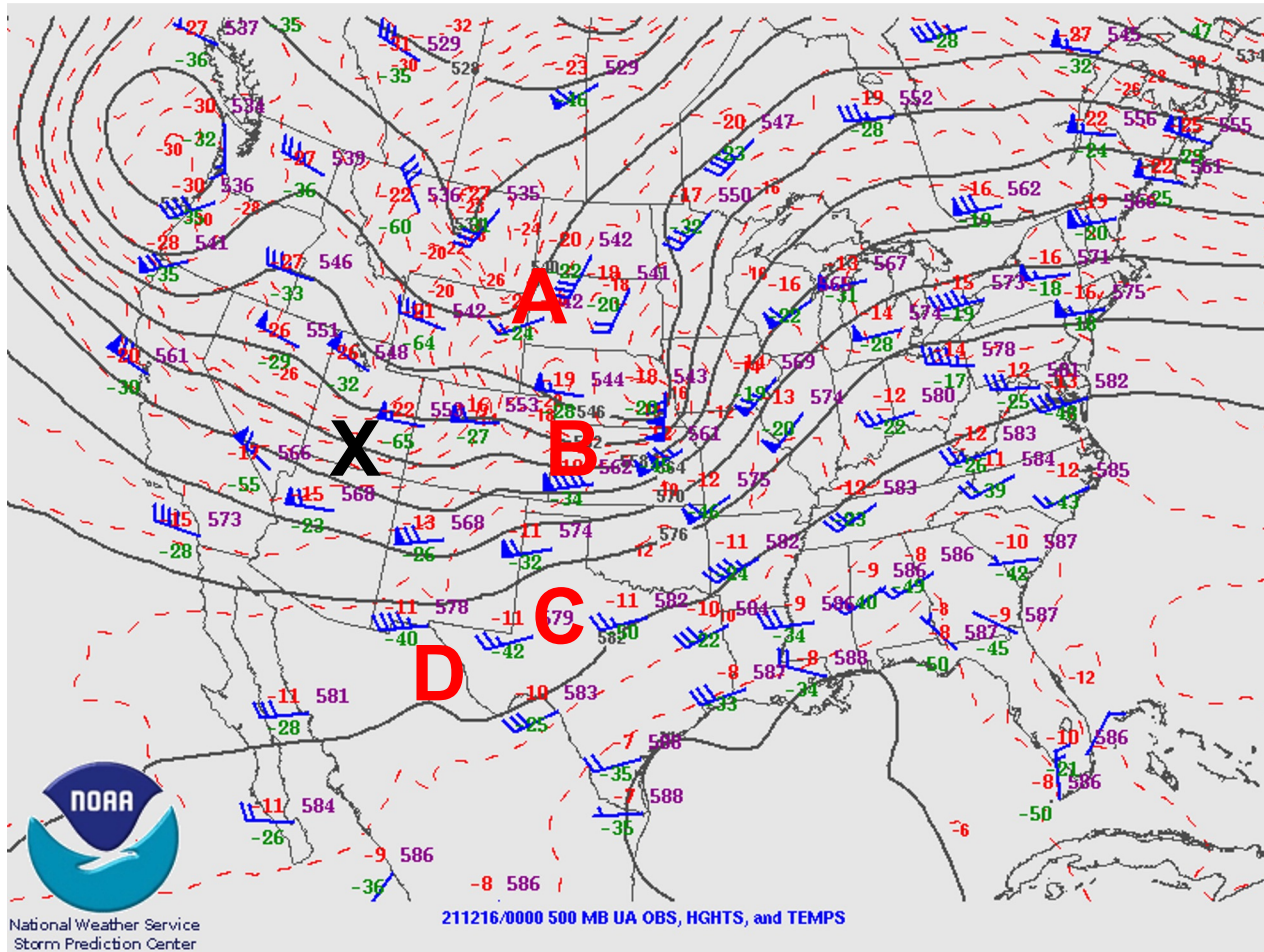


How about this case?

Where do you think this trough will go in the next 12 hours?

(Choose A, B, C, or D)

QG X Examples



How about this case?

Where do you think this trough will go in the next 12 hours?

(Choose A, B, C, or D)

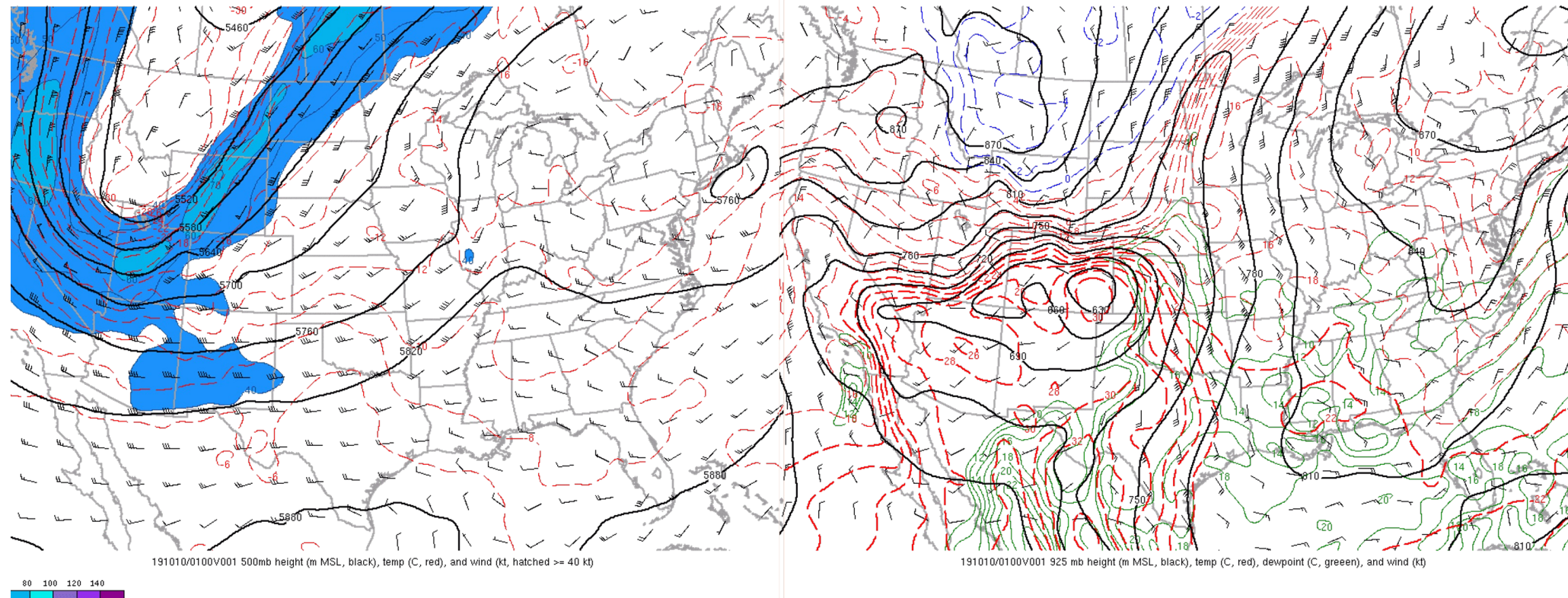
QG X Examples

Watch how the 500 mb trough deepens as the 925 mb cold front surges south into the Plains. This is an example of differential thermal advection.

/NWS/Storm Prediction Center

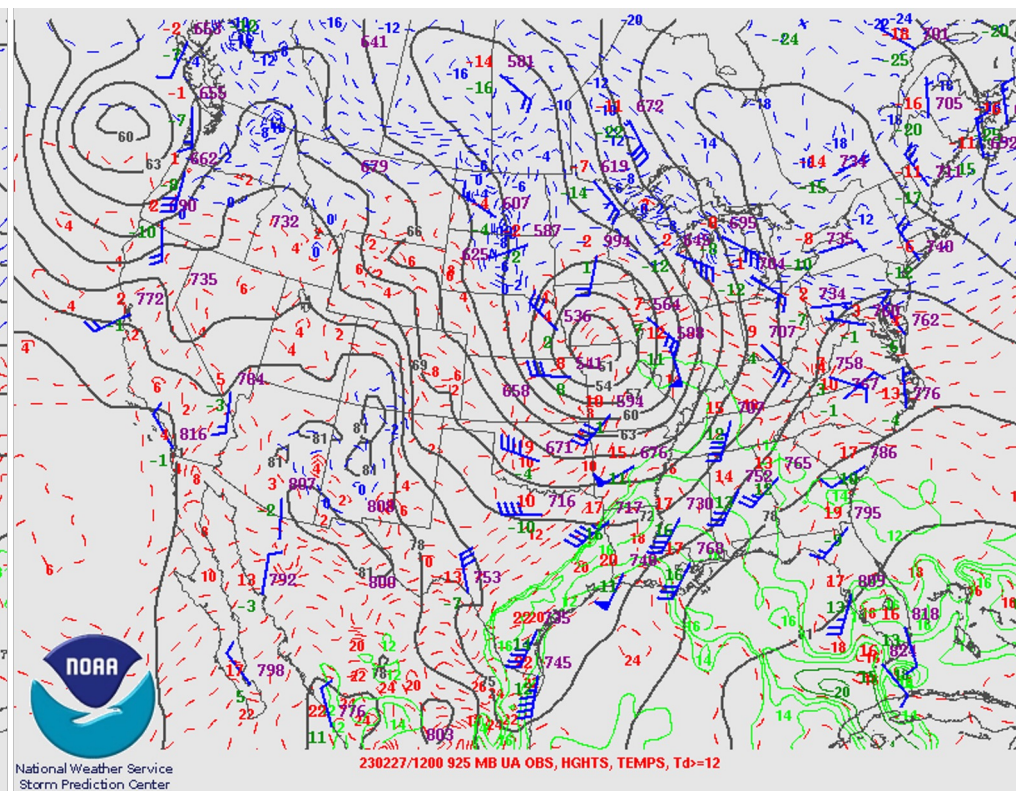
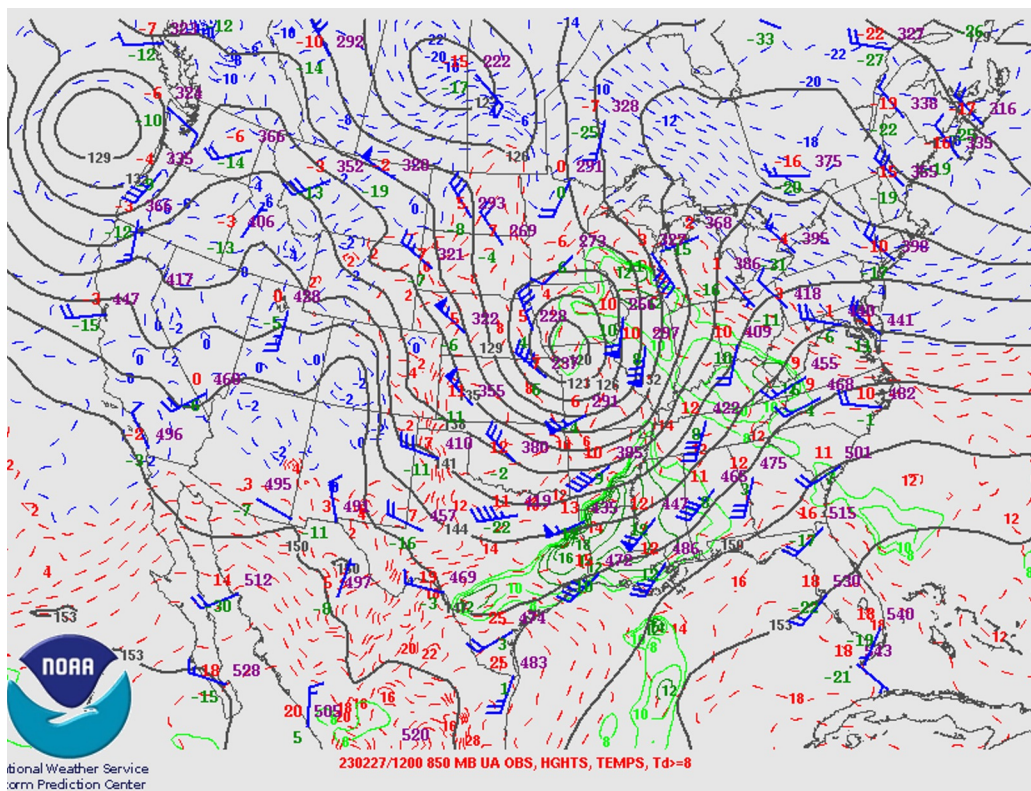
Mesoscale Analysis Data NOAA/NWS/Storm Prediction Center

Mesoscale Analysis



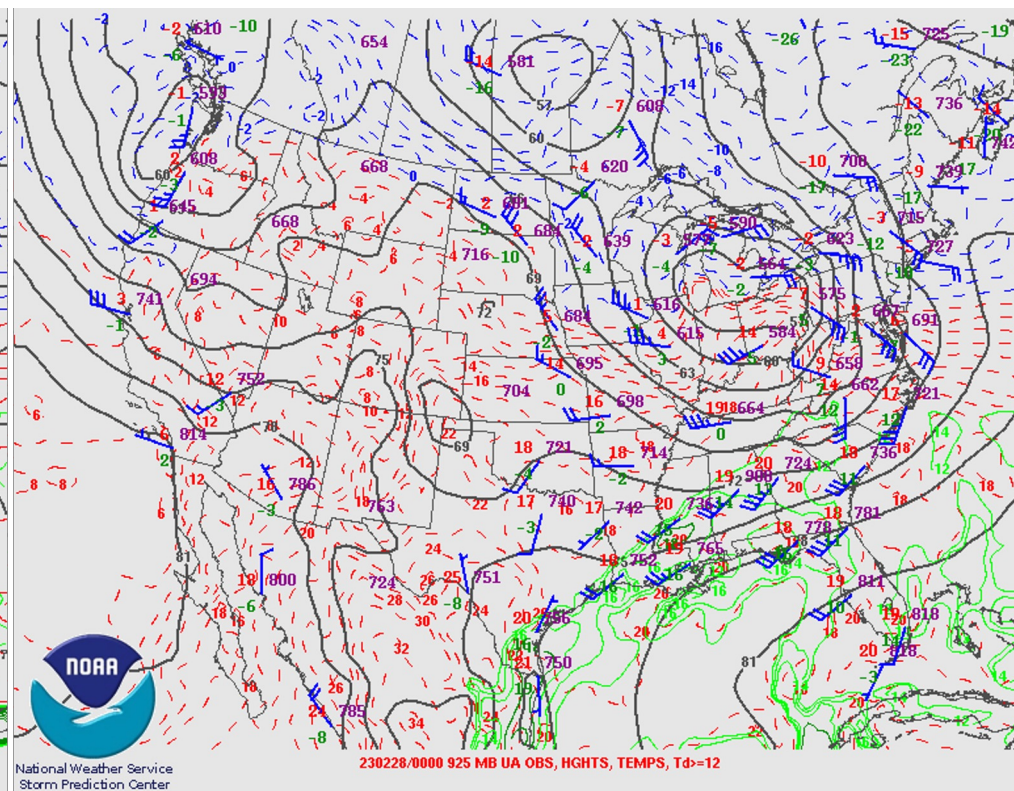
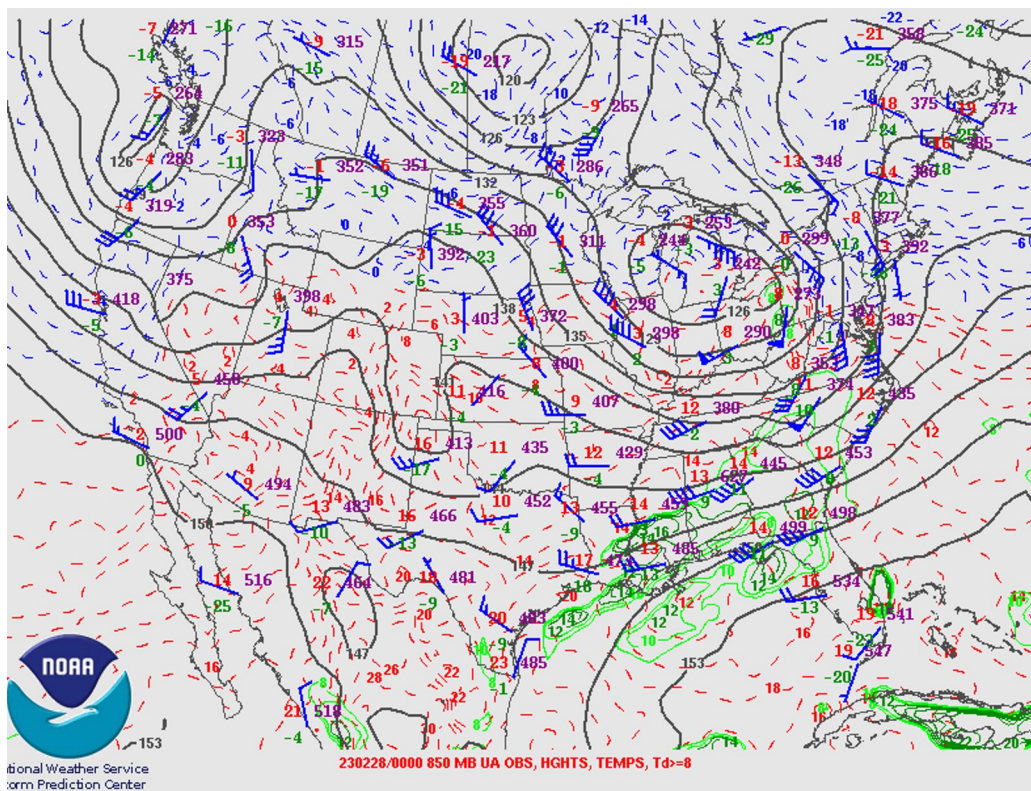
QG X Examples

Where is the strongest warm air advection at 850 mb?
This will tell you where the 925 mb low should go!



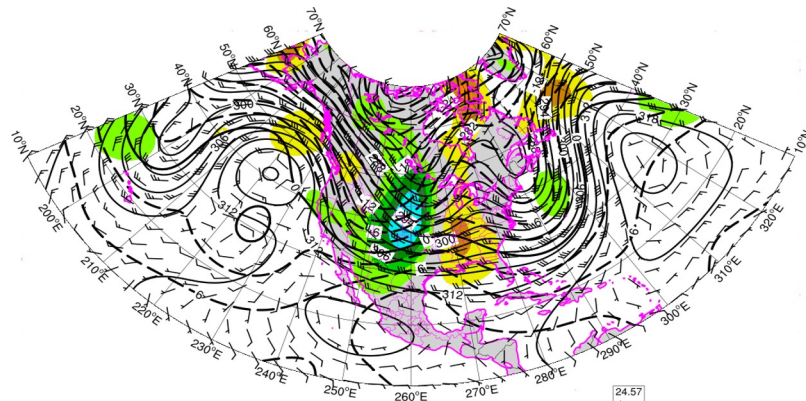
QG X Examples

Where is the strongest warm air advection at 850 mb?
This will tell you where the 925 mb low should go!

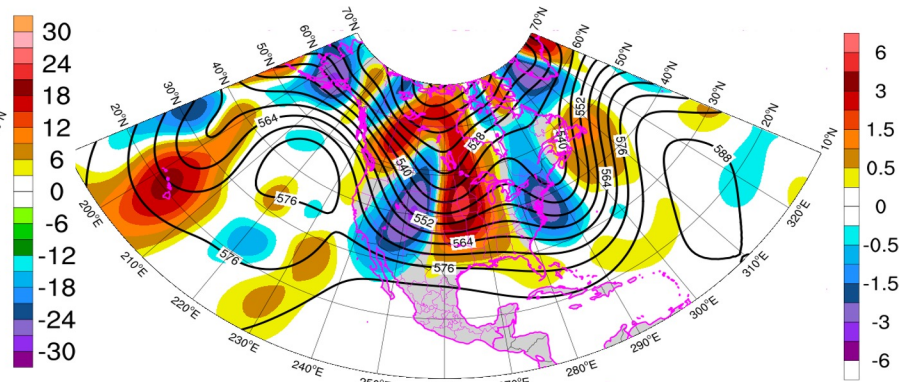


QG Resources

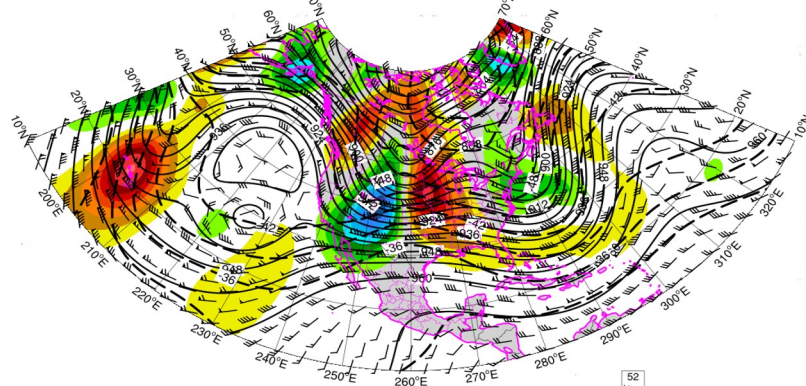
<https://inside.nssl.noaa.gov/tgalarneau/real-time-qg-diagnostics/>



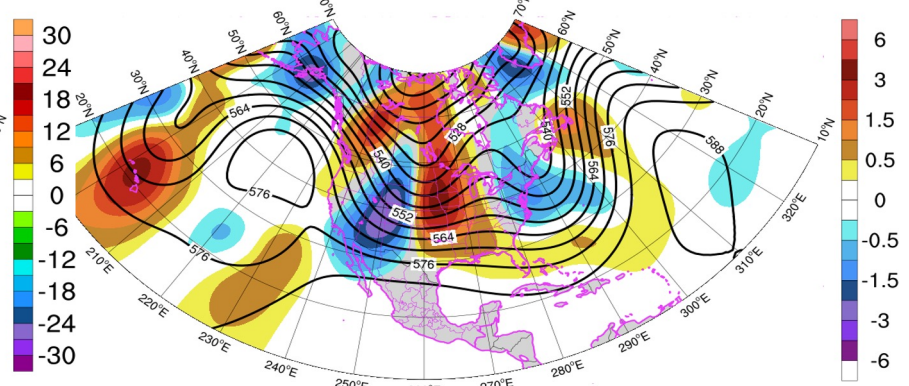
700 hPa Height, Temperature, Geostrophic Wind, and Temperature Advection (smoothed)
0-h CMC Global forecast at 2021020412



500 hPa Z and Total RHS QG Z Tendency Forcing (traditional form) (smoothed)
0-h CMC Global forecast at 2021020412



300 hPa Height, Temperature, Geostrophic Wind, and Temperature Advection (smoothed)
0-h CMC Global forecast at 2021020412



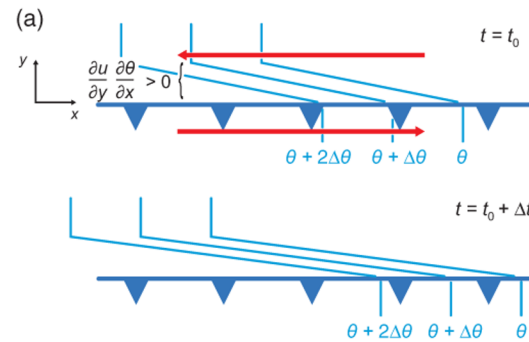
500 hPa Z and Amplification Term (B) QG Z Tendency Forcing (traditional form) (smoothed)
0-h CMC Global forecast at 2021020412

Frontogenesis

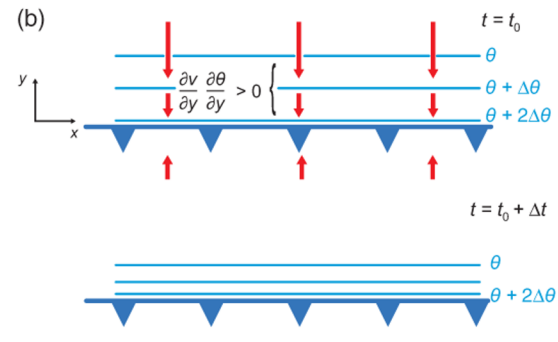
- Frontogenesis (frontolysis) is the strengthening (weakening) of a temperature gradient
- In case of frontogenesis, thermal wind balance (TWB) is violated because temperature gradient is too strong for the given wind shear
 - To restore TWB, atmosphere weakens temperature gradient via ascent (adiabatic cooling) on warm side and descent (adiabatic warming) on cold side of gradient
- Fronts are zones where thermal advection and frontogenesis are easily enhanced and are also preferred corridors for cyclones/cyclogenesis

Frontogenesis Mechanisms

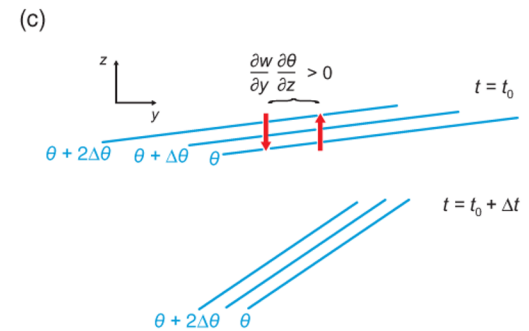
horizontal shear



confluence



tilting



Differential
adiabatic heating

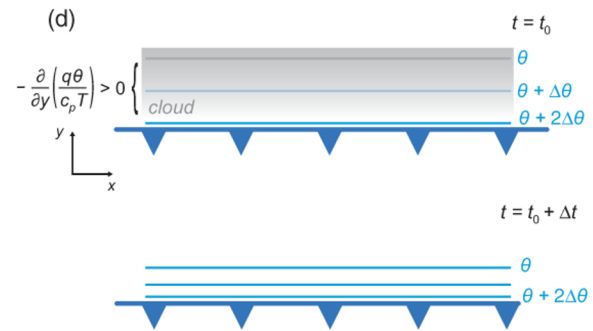


Fig. 5.4 MR

Frontogenesis and Frontolysis by Confluence

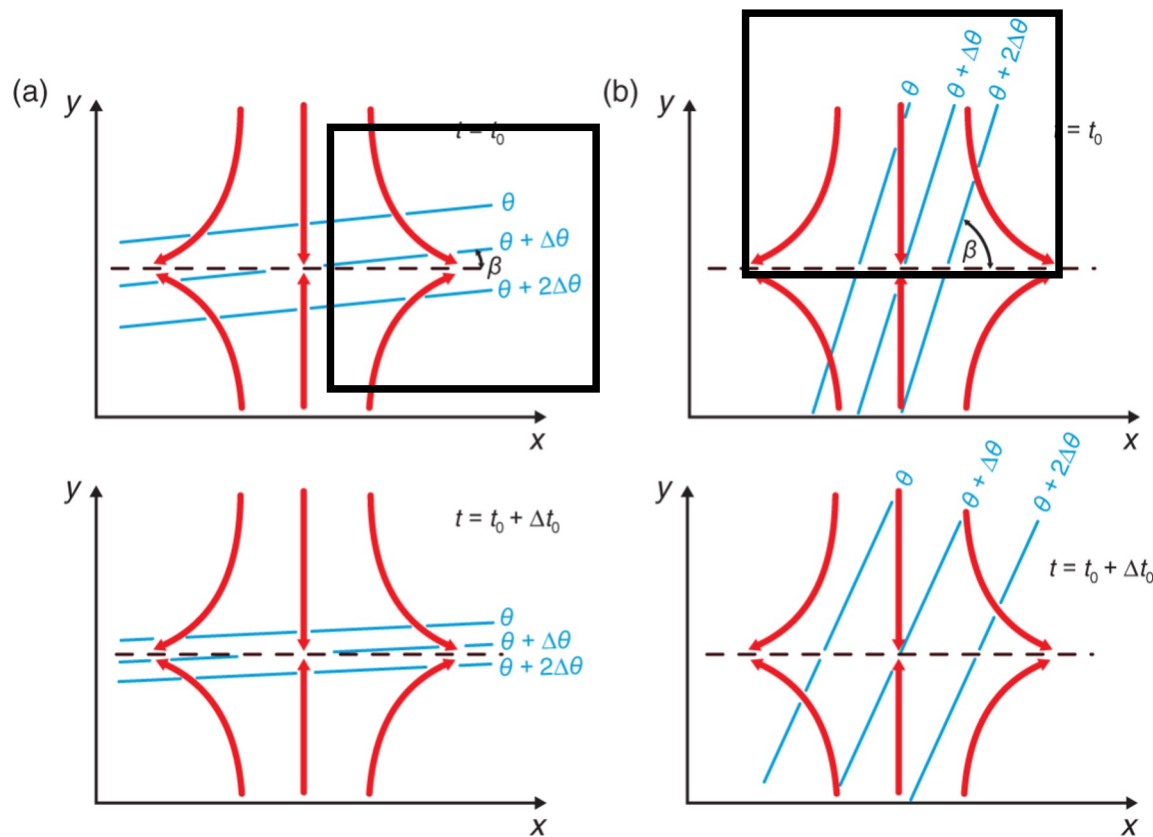
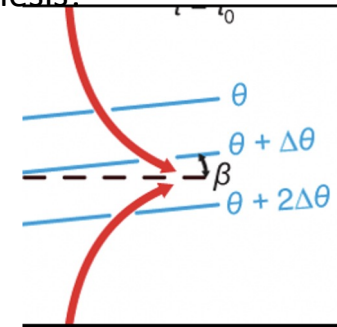
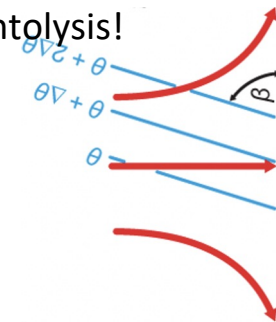


Fig. 5.5 MR
2010

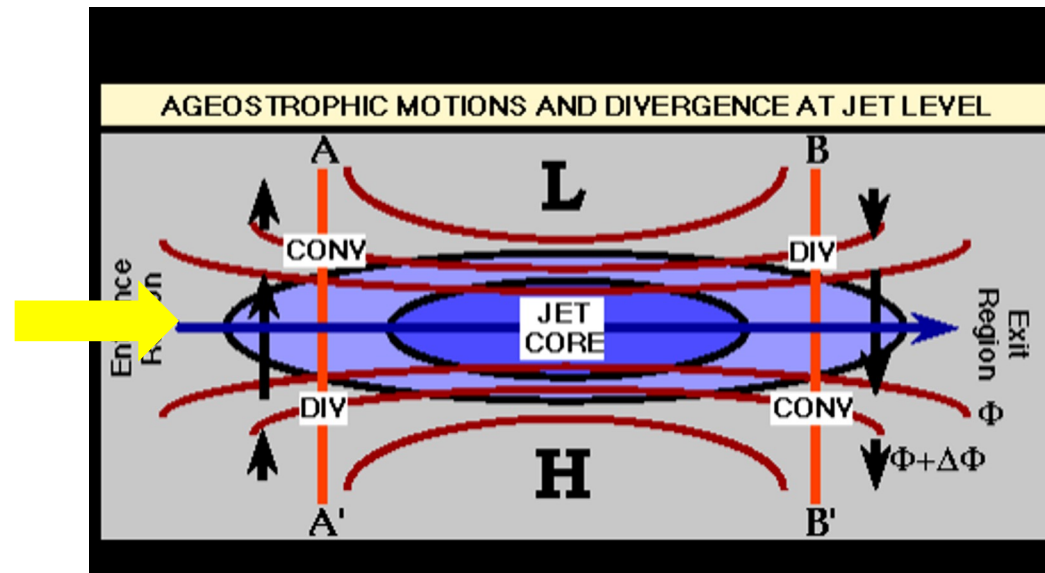
Like a jet entrance region:
Frontogenesis!



Like a jet exit region:
Frontolysis!



Frontogenesis and Jet Streaks



Air entering jet streak – sinking on cold side, rising on warm side.

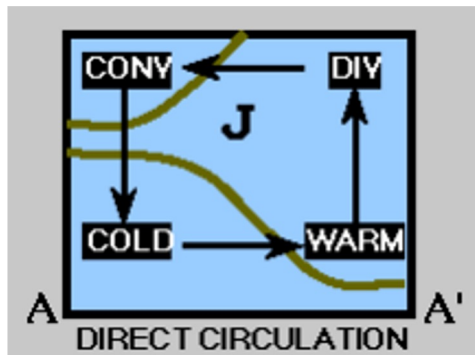
Air leaving jet streak – rising on cold side, sinking on warm side.

Vertical Wind Shear

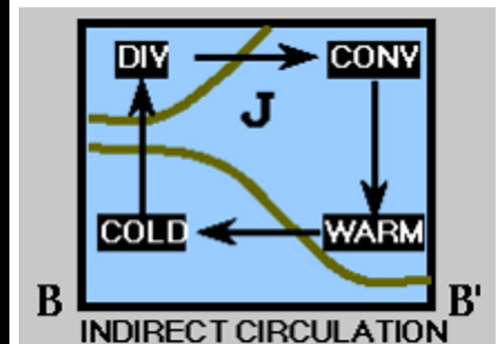
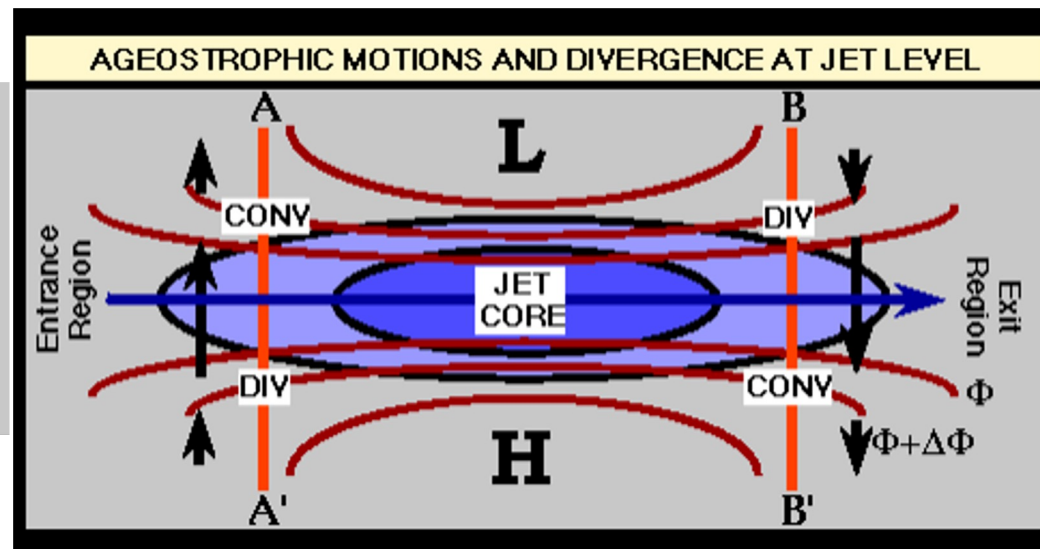
Where does it come from?

Secondary contributions:

Large accelerations of the horizontal wind due to large ageostrophic winds
(think near jet streaks, areas of frontogenesis, and/or rapidly intensifying cyclones).



Erodes horizontal
temperature gradient
(weaker thermal wind)



Enhances horizontal
temperature gradient
(stronger thermal wind!)

For additional reading: M.R. 2010 and Doswell

Frontogenesis and Jet Streaks

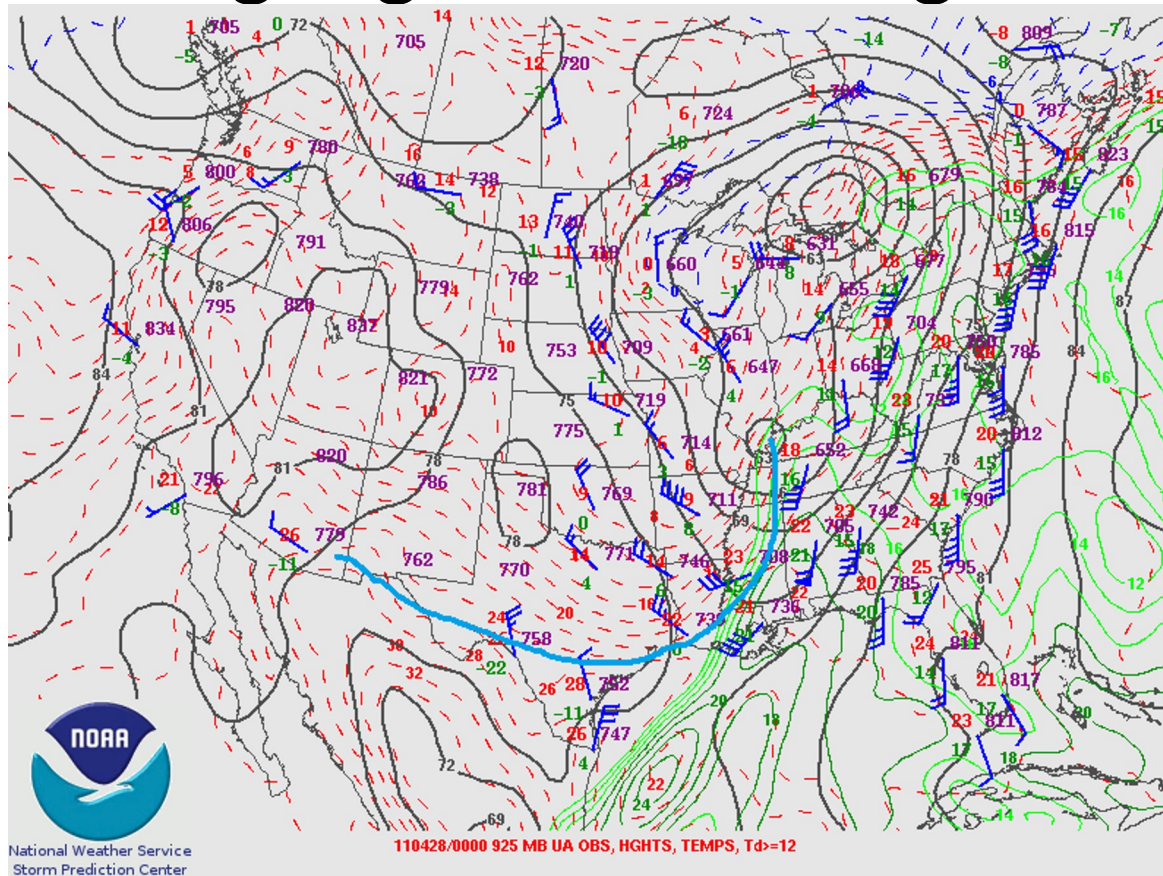
- Jet streaks are coincident with strong temperature gradients (thermal wind balance!)
- Air flows through a jet streak
 - Encounters strengthening temperature gradient (frontogenesis) in entrance region
 - Encounters weakening temperature gradient (frontolysis) in exit region
- Response to frontogenesis in entrance region is ascent on warm side (right entrance) and descent on cold side (left entrance)
- Response to frontolysis in exit region is?

Baroclinic systems

- Vorticity and thermal structure tilts westward (upstream) with height
 - Deepening/strengthening systems
 - Differential thermal advection leads to destabilization
- Warm advection corresponds to veering winds with height
 - Large clockwise turning hodographs in warm sector
- Strong jet streaks and fronts are present

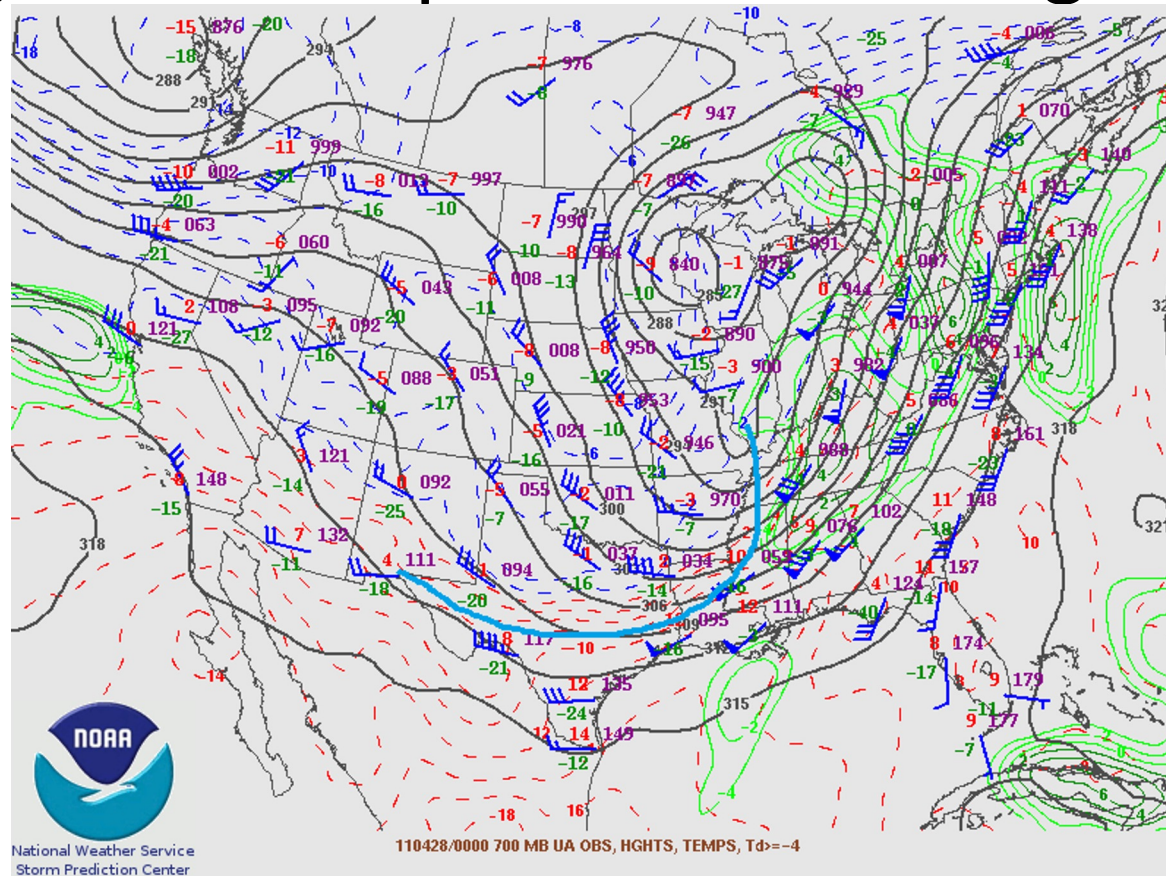
Edge of stronger gradient near ground

925m
b

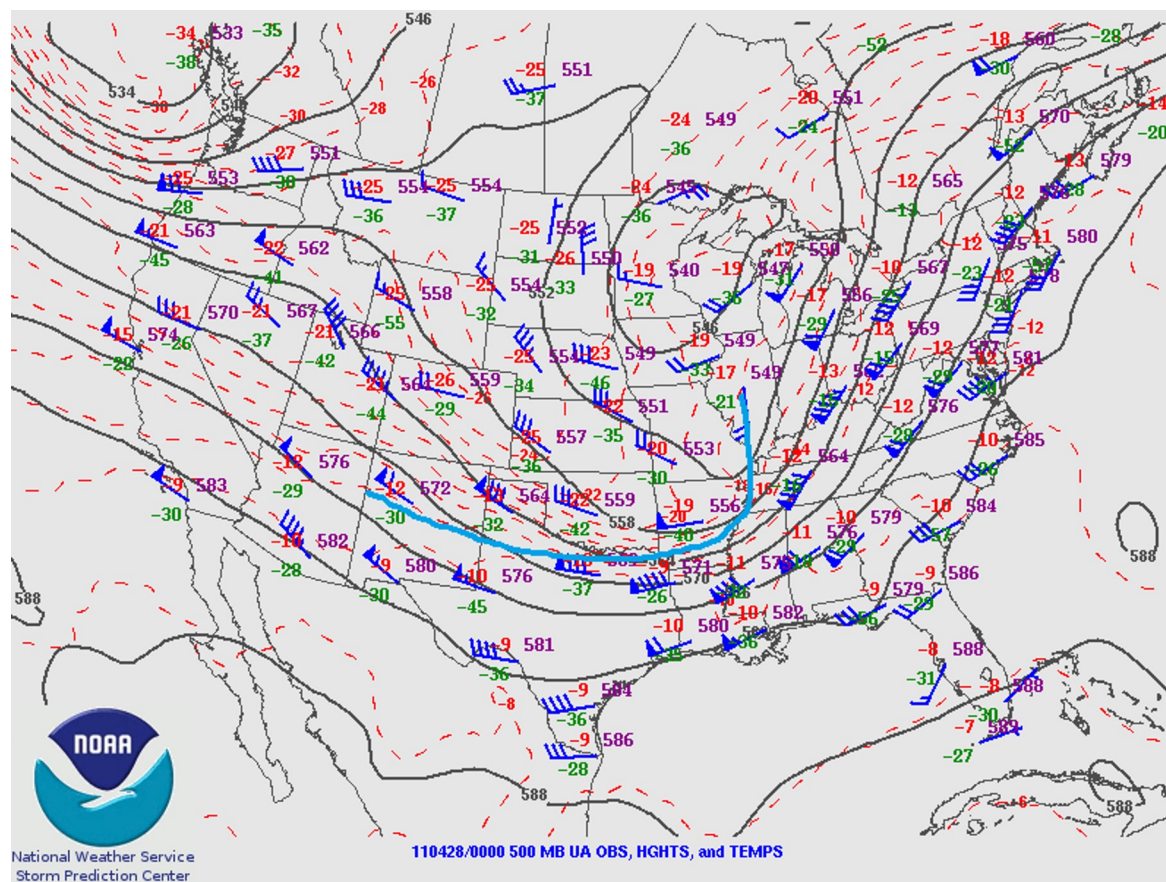


Temp gradient slopes NW with height

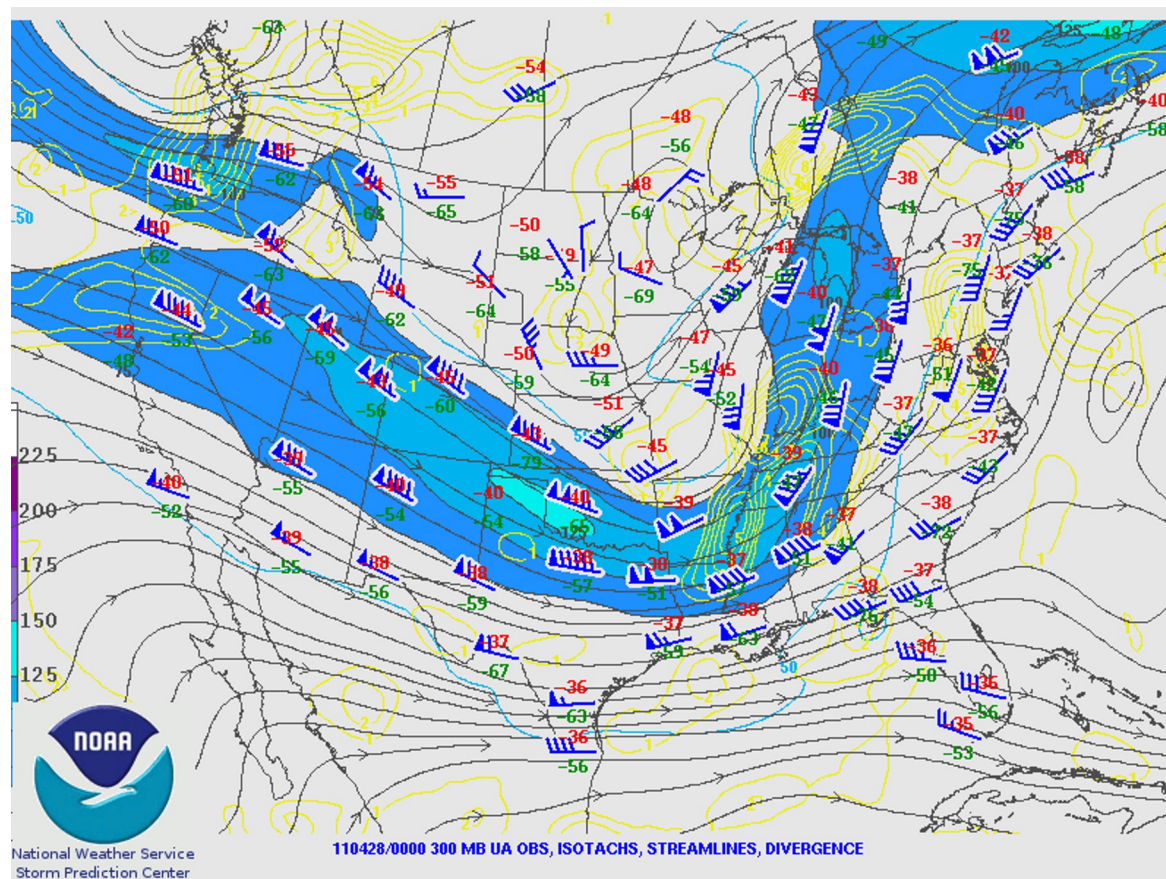
700m
b



500m
b

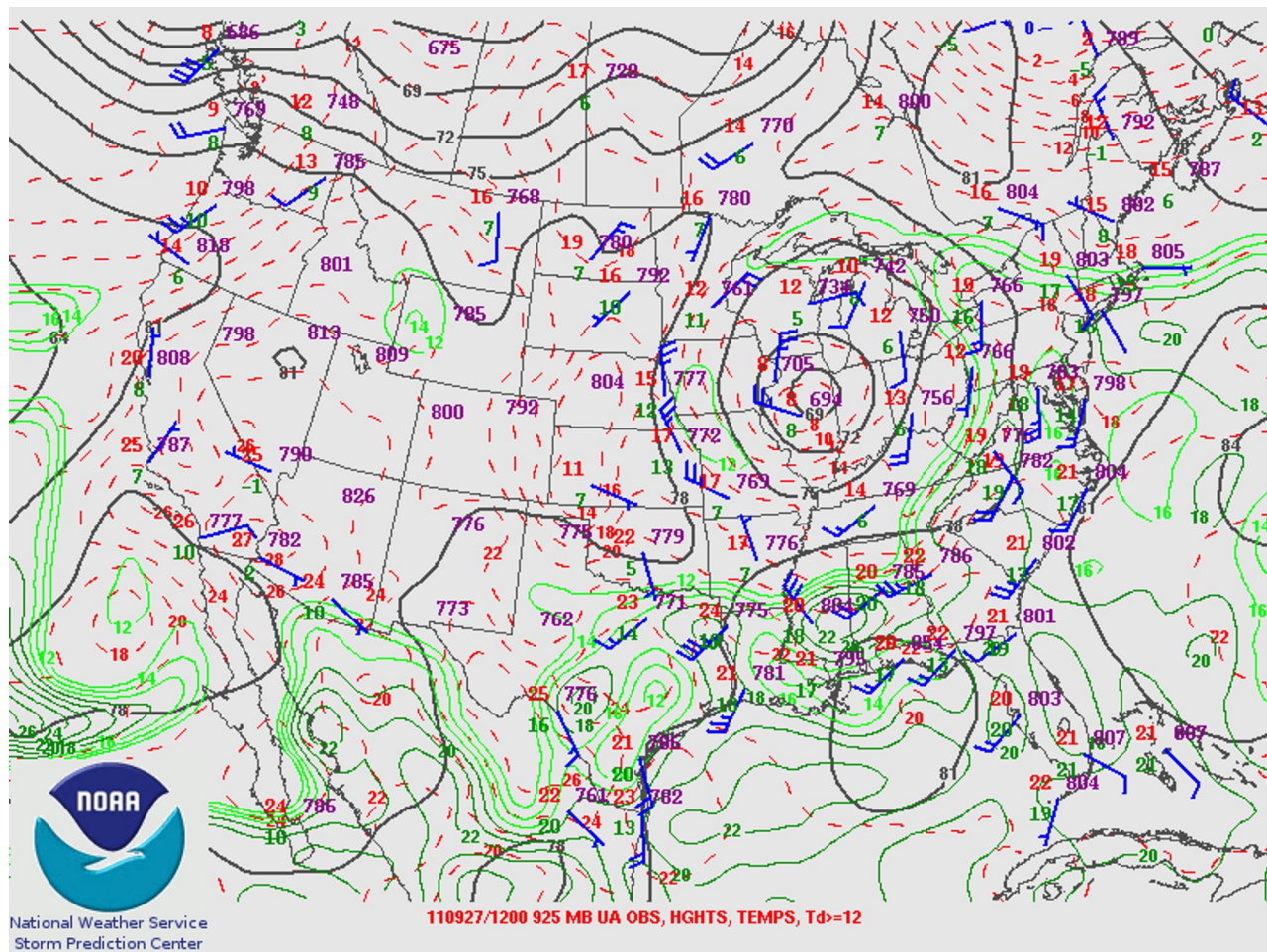


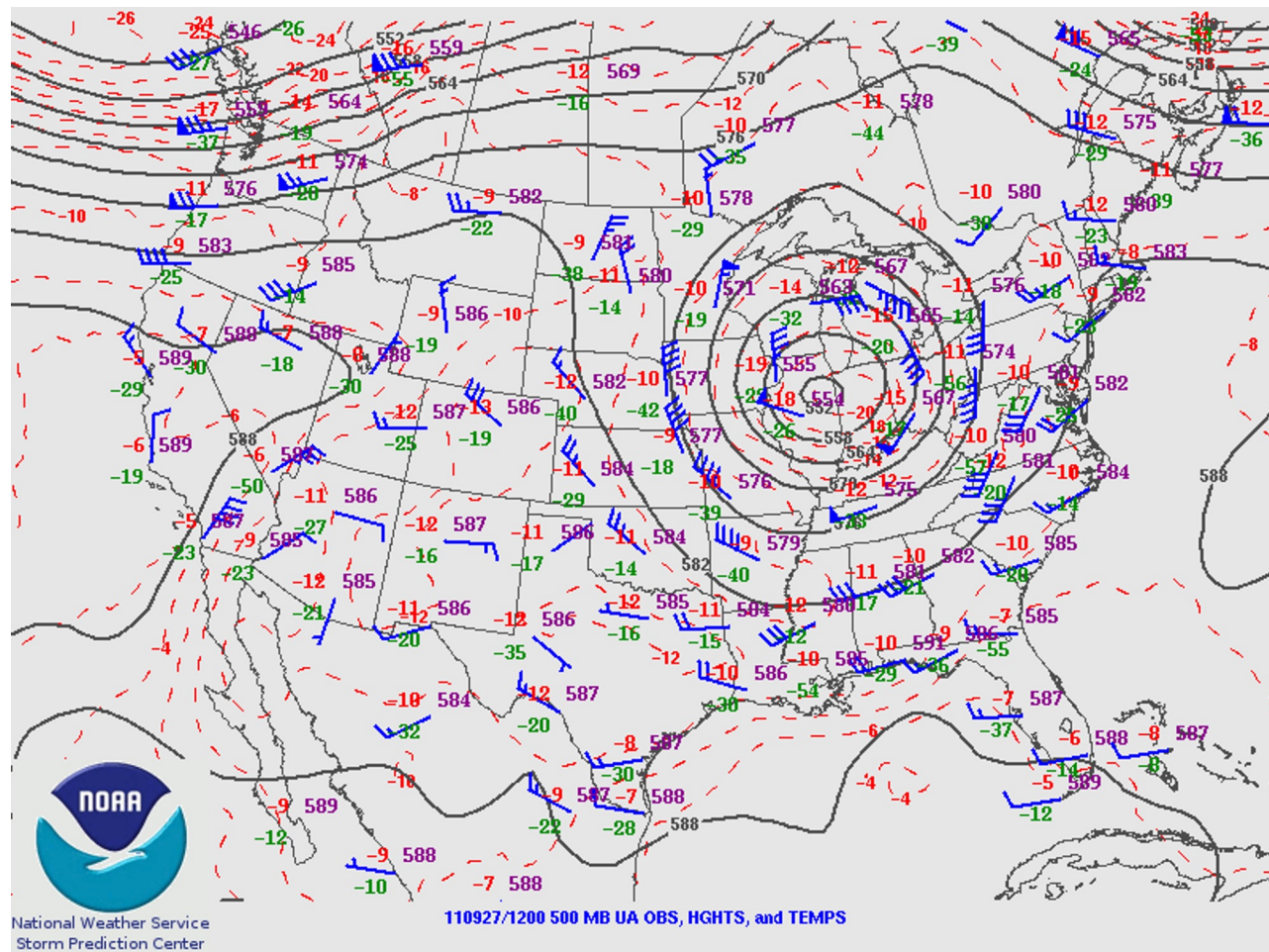
300m
b



Equivalent Barotropic Systems

- Vorticity and thermal structure vertically stacked with height
 - Steady state or weakening systems
- Minimal temperature advection corresponds to weak vertical shear
 - No feedback mechanism to intensify.
- Weak gradients – weak flow – slow moving
 - Can even retrograde if large enough because planetary vorticity begins to dominate the relative component.





Final Comments

- Can explain development of weather systems (QG height tendency and QG cyclogenesis)
- Can explain why ascent occurs where it does near troughs, jets, and fronts
- Look for gradients!
- Synoptic-scale processes set the stage for severe thunderstorm development